創新
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精緻教學

2025年
台灣數學教育學會年會
暨第十七屆
科技與數學教育國際學術研討會

日期 2025/05/17(Sat.)-2025/05/18(Sun.)

地點 國立臺中教育大學

目錄

大會組織

指導單位：

國立臺中教育大學 數學教育學系
台灣數學教育學會
國立臺中教育大學師培處暨數學教學領域研究中心
國立臺北教育大學 數學暨資訊教育學系

大會主席：

國立臺中教育大學校長 郭伯臣
國立臺北教育大學校長 陳慶和

大會副主席 (依姓氏筆劃排序)：

楊凱琳教授 (台灣數學教育學會理事長)
黃國禎教授 (國立臺中教育大學副校長)
陳錦章教授 (國立臺中教育大學理學院院長)
林素微教授 (科技部數學教育學門召集人)

議程主席：

袁 媛教授 (國立臺中教育大學 數學教育學系系主任)

議程委員（依姓氏筆劃排序）：

王婷瑩教授 左台益教授 吳正新教授 吳昭容教授 李梓楠教授 李林滄教授
李源順教授 林勇吉教授 林原宏教授 林碧珍教授 林素微教授 姚如芬教授
洪文良教授 胡豐榮教授 英家銘教授 徐偉民教授 張育萍教授 張淑怡教授
許慧玉教授 陳中川教授 陳建誠教授 陳致澄教授 陳嘉皇教授 陳錦章教授
單維彰教授 黃馨瑩教授 游自達教授 黃國禎教授 楊晉民教授 楊凱琳教授
詹勳國教授 劉柏宏教授 劉宣谷教授 鄭英豪教授 鄭博文教授 賴冠州教授
謝佳叡教授 謝豐瑞教授 謝闓如教授 魏士軒教授 蘇意雯教授

2025 年台灣數學教育學會年會暨第 17 屆科技與數學教育國際學術研討會

時間：2025.05.17-2025.05.18 地點：國立臺中教育大學

2025.05.17 (星期六)

時間	項目	地點	主持人
08:00-08:30	報到	大廳 K108	行政組
08:30-08:40	研討會開幕(校長來賓致詞)	音樂廳 K101	袁媛教授
08:40-09:30	**特邀演講 I** **主講者**：Jinfa Cai Professor, University of Delaware, USA **講題**：Supporting Teachers to Teach Mathematics Through Problem Posing	音樂廳 K101	單維彰教授
09:40-10:20	**【林福來教授博碩士論文獎頒獎暨獲獎人論文主題報告】** 博士學位論文獎(從缺) 碩士學位論文獎 頒獎人：林福來教授	音樂廳 K101	楊凱琳教授
10:20-11:20	茶敘暨海報展示	大廳 K108	行政組
10:30-11:20	**TI 工作坊(I)** **主講者**： Siew Tong Lee School of Art, Singapore **講題**：圖形計算機技術在統計學的應用	數學樓 3 樓 C301	李明恭教授
11:20-12:10	**TI 工作坊（II）** **主講者**：李睿紘 T3 Instructor **講題**：運用圖形計算機科技理解變換的意義	數學樓 3 樓 C301	李明恭教授
11:20-12:10	台灣數學教育學會 第九屆第 2 次會員大會	音樂廳 K101	楊凱琳教授
12:20-13:10	台灣數學教育學會 第九屆第 4 次理監事會議	求真樓 4 樓 K401	楊凱琳教授
12:20-13:40	午餐	大廳 K108	行政組
13:40-15:20	數學教材教法教授們與師資生論壇 (大學教授與師資生場次)	演講廳 K107	主持人: 李源順教授 與談人: 林原宏教授 姚如芬教授 單維彰教授
13:40-15:20	**論文發表場次一** AI 語言處理與情緒運算在數學學習行為測量上之應用 偏鄉教師專業學習社群融入 JDM 教材之行動研究	數學樓 3 樓 C301	楊凱琳教授

	數學史融入國小教學-九章算術圓田		
15:20-15:40	茶敘	大廳 K108	行政組
15:40-16:30	**專題演講** 主講者：Siew Tong Lee School of Art, Singapore 講題：圖形計算機技術在微積分教學的應用	演講廳 K107	Eric Chin
16:30-17:20	**專題演講** 主講者：Eric Chin Education Technology Consultant, Singapore 講題：STEM 之教育探索篇	演講廳 K107	Siew Tong Lee
15:40-17:20	**論文發表場次二**	數學樓 3 樓 C301	謝佳叡教授
	國中數學教科書試題之比較與分析 一以二元一次聯立方程式為例		
	臺灣與中國國小數學教科書在分數不同意義的數學活動類型之比較		
	臺灣高中數學教師對導數概念的認識論觀點		
	論文發表場次三	數學樓 3 樓 C302	李源順教授
	國小五年級視知覺及操弄之個案研究		
	國小六年級擬題的數學創造力表現之研究		
	結合提問和數學感提升學生文字題解題表現之個案研究		
	論文發表場次四	數學樓 3 樓 C303	張育萍教授
	八年級學生經歷數位數學遊戲學習等差數列的感受		
	數學教育相關研究結果在數學課堂實踐情況的調查研究： 以差異化教學為例		
	數學奠基模組融入國小六年級圓與扇形面積教學之個案研究		
	論文發表場次五	數學樓 3 樓 C304	魏士軒教授
	六年級個案學生在比與比值及其相關單元的解題信念		
	國小六年級學生錯誤類型之研究 —縮圖、放大圖與比例尺為例		
	提升四年級學習扶助學生文字題解題能力之個案研究		

2025 年台灣數學教育學會年會暨第 17 屆科技與數學教育國際學術研討會

2025.05.18 (星期日)

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<tr><td>09:10-10:00</td><td>特邀演講 II
主講者：Charalambos Y. Charalambous
Associate Professor, University of Cyprus
講題：Studying and Understanding Teaching Quality in Mathematics: Advancements Made and Challenges Faced</td><td>音樂廳 K101</td><td>許慧玉教授</td></tr>
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<tr><td>10:50-11:40</td><td>特邀演講 III
主講者：Lynda Ball
Senior lecturer, University of Melbourne, Australia
講題：Technology for Teaching and Learning Mathematics: Opportunities and Issues</td><td>音樂廳 K101</td><td>楊凱琳教授</td></tr>
<tr><td>11:40-13:30</td><td>午餐</td><td>大廳 K108</td><td>行政組</td></tr>
<tr><td rowspan="16">13:30-14:30</td><td>論文發表場次六</td><td rowspan="4">數學樓 3 樓
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<tr><td>運用 Polya 解題策略教學降低五年級學生數學學習焦慮之行動研究-以臺中市某國小為例</td></tr>
<tr><td>運用數學寫作於國小五年級整數四則計算之行動研究</td></tr>
<tr><td>國小高年級學生數學情緒初探</td></tr>
<tr><td>論文發表場次七</td><td rowspan="4">數學樓 3 樓
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<tr><td>以《圖騰島》桌遊探討小學生空間推理能力之促進效果</td></tr>
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<tr><td>診斷教學應用於數學分數單元教學成效之研究</td></tr>
<tr><td>異分母分數加減的迷思概念探究與教學設計</td></tr>
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<tr><td>學生影片製作與討論促進數學參與：比較 YouTuber 型式與教學型式任務設計</td></tr>
<tr><td>因倍數概念之學習診斷工具開發之前導研究</td></tr>
<tr><td>應用三階段評量探討國小五年級學童在因倍數問題的解題表現</td></tr>
<tr><td rowspan="3">14:40-15:40</td><td>論文發表場次十</td><td rowspan="3">數學樓 3 樓
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<tr><td>差異化教學中的彈性分組策略對國小六年級學生數學學習成效之影響</td></tr>
<tr><td>探究教師在數學數位差異化教學中轉變之理論框架</td></tr>
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時間	項目	地點	主持人
	論文發表場次十一	數學樓 3 樓 C302	謝闓如教授
	國小二年級低成就學童數學外加課程舉例能力之教學初探		
	國中數學課後學習扶助使用數位學習平台之個案研究		
	學生小組成就區分合作學習融入六年級小數與分數的計算教學之研究		
	論文發表場次十二	數學樓 3 樓 C304	楊晉民教授
	以數學感「舉例」策略融入一年級數的單元之初探		
	IMPROVE 後設認知教學法對國小學生數學學習影響初探		

2025 年第十七屆
科技與數學教育國際學術研討會
暨數學教學工作坊

Proceeding of 2025 The Seventeenth International

Conference on Technology and Mathematics Education

and Workshop of Mathematics Teaching

特邀演講

主講者：

Jinfa Cai Professor

Charalambos Y. Charalambous Associate Professor

Lynda Ball Senior lecturer

What Research Says About Problem Posing Based Learning (P-PBL)

Jinfa Cai
University of Delaware

jcai@udel.edu

Supporting Teachers to Teach Mathematics Through Problem Posing: An Early-Stage Longitudinal Study (in Middle Grades)

Many Thanks
to
Professors Yuan and Lin
for the Invitation

It is my 6th time to TW

Thanks to My Team

- Jinfa Cai, PI and Director
- Faith Muirhead & Amy Brown, co-PIs (Michelle Cirillo)
- Stephen Hwang, Senior research associate and project coordinator
- J.P. Han, Burak ölmez, & Hua Ran, postdocs
- Victoria Robinson, research specialist
- Yue Ma, Matt Melville, Steve Silber, GRAs
- Simon Glover, Jenna Paltenstein, Undergraduate RAs

Outline

1. Why Problem Posing?
2. What does teaching mathematics through problem posing look like?
3. What is problem posing, anyway?
4. What is a problem-posing task?
5. How should teachers handle students’ posed problems in classroom instruction?
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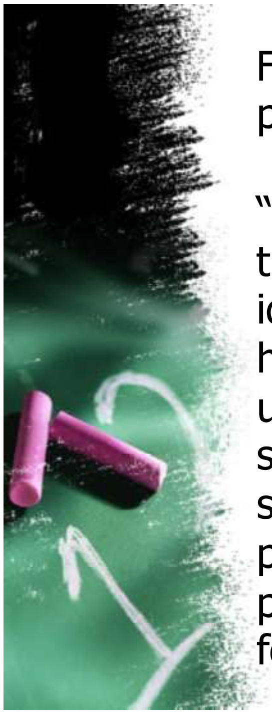
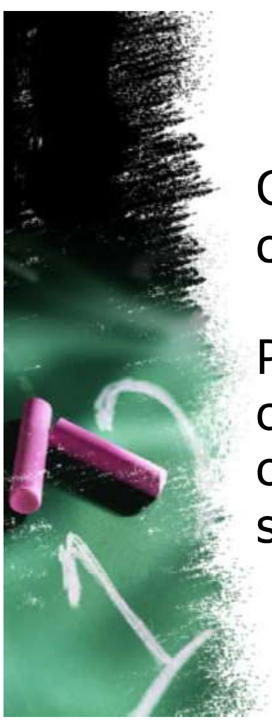


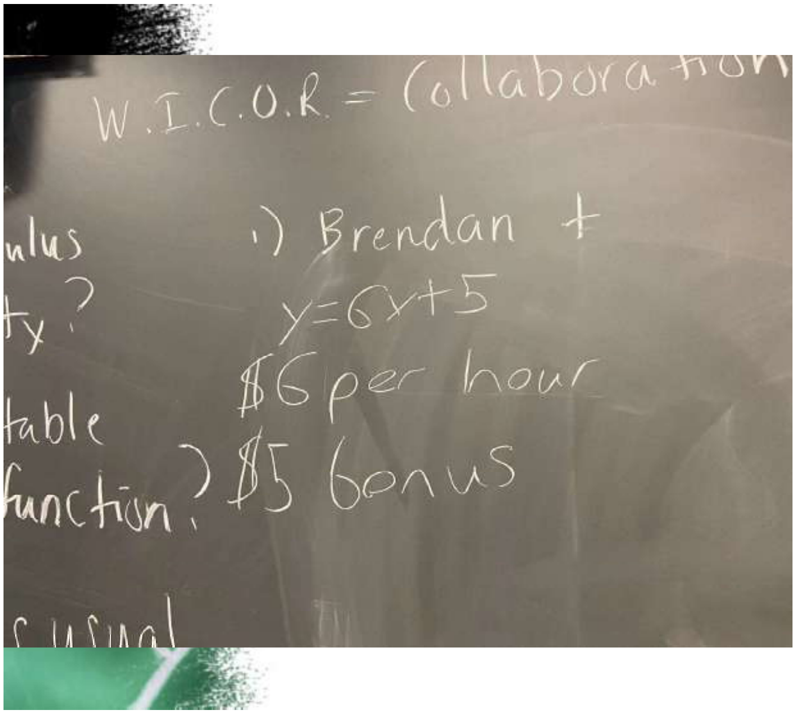
For the linear Function y = 6x + 5, list two ordered pairs.

"The lesson did not go all that well. While most of the students were able to plot points, many had no idea why the "order" in ordered-pair matters. They had never seen quadrants, and many did not understand how to answer the question regarding the signs of the ordered pairs in the quadrants and no student realized that there is a zero involved if the point is on an axis. The idea of producing ordered pairs for a function was beyond the grasp of all but a few of the students. MUCH WORK TO DO!"

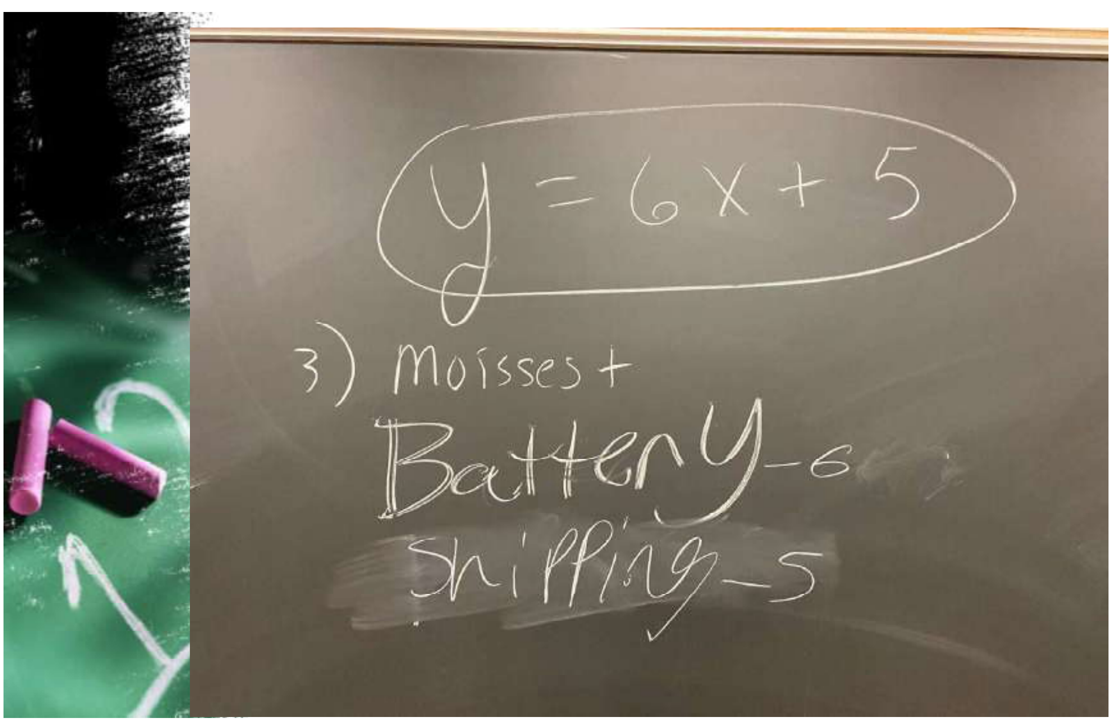


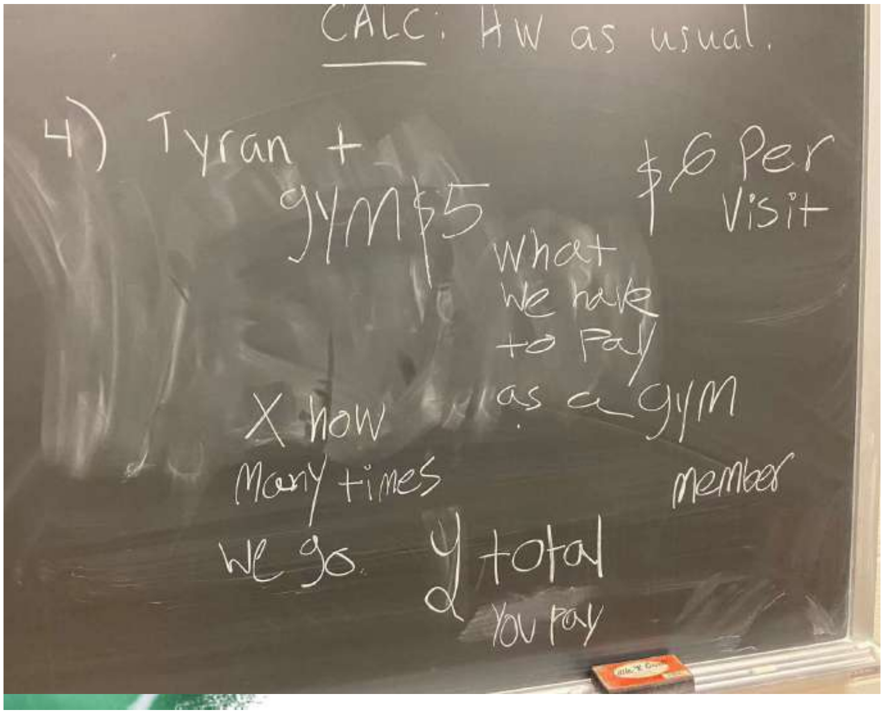
Given the linear function, y = 6x + 5, what could this represent a real-life scenario?

Pose an easy mathematical problem and a challenging mathematical problem which can be solved based on your real-life scenario.



Y = 6x + 5
$6 per hour
$5 bonus





Gym, $5
What we have to pay as a gym member

$6 per visit

X: how many times you go

Y: total you pay

Proportion of Mathematical Problems for Each Task (Silber & Cai 2021)

	Task 1	Task 2	Task 3	Task 4
High Performing (Grades A&B) (n = 57)	89%	100%	98%	96%
Average Performing (Grade C) (n = 36)	100%	97%	100%	100%
Low Performing (Grades D&F) (n = 42)	69%	95%	100%	95%

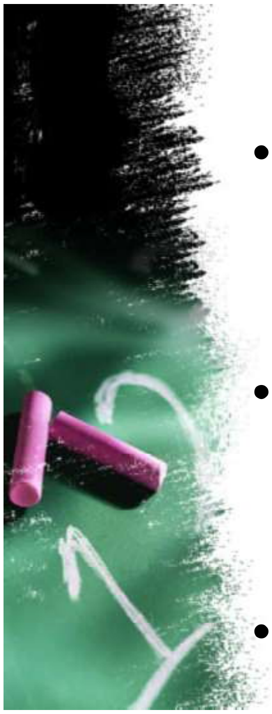
Proportion of Responses That Invoked Mathematical Ideas for the Graph Task

	Non-zero y-intercept	Covariation	½ as the slope
High Performing	43%	64%	14%
Average Performing	33%	50%	0%
Low Performing	26%	63%	16%

Students’ Reactions

- I think [problem posing] gives a new perspective. It makes you think about it in a different way, makes you more familiar with the mathematics principles behind the questions.
- You’re looking at the entirety of the information given, whereas if you’re given a question, you’re only going to give it a quick glance for what’s relevant to what it’s asking. You might not understand the big picture of how the problem works. Whereas if we [pose problems], you’re sort of seeing the entirety of it.

Silber & Cai, 2024



Why Problem Posing?

- Theoretically, P-PBL is sound based on both constructivist and sociocultural perspectives on learning, and
- It can increase students’ access (or opportunity) to mathematical sensemaking and learning.
- Low floor high ceiling

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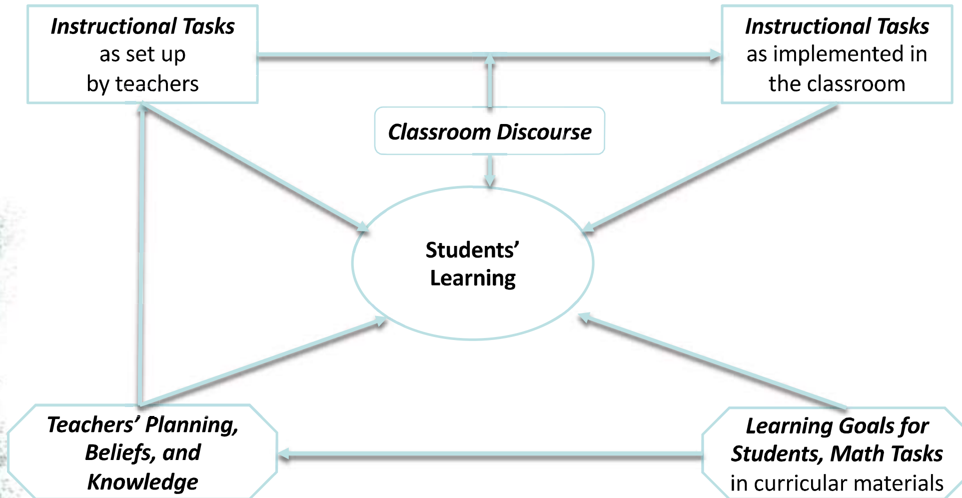
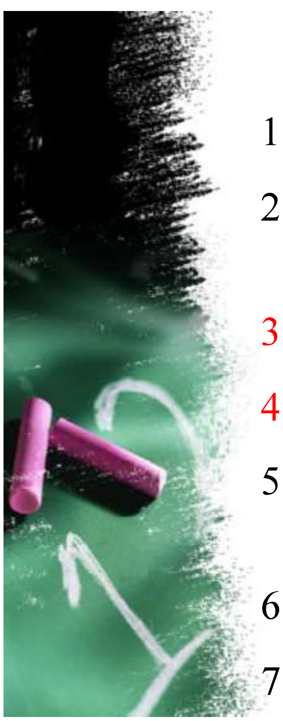
Teaching Mathematics Through Problem Posing



With the following numbers and symbol, please pose one easy problem and one challenging problem involving percentages.

40 50 %

- This task is used to review the unit of percent
- Students pose problems individually
- Students present their posed problems
- Teacher writes down students' posed problems on the board in different categories (Park & Leinhardt, 1995)
- Teacher and students solve selected problems



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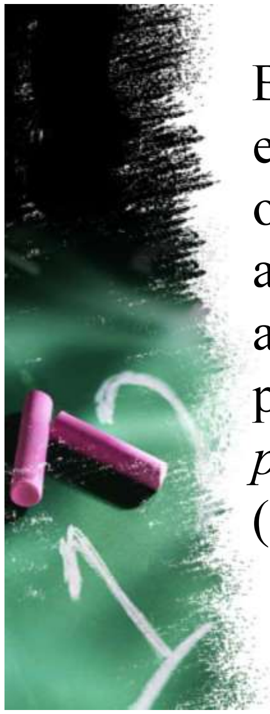
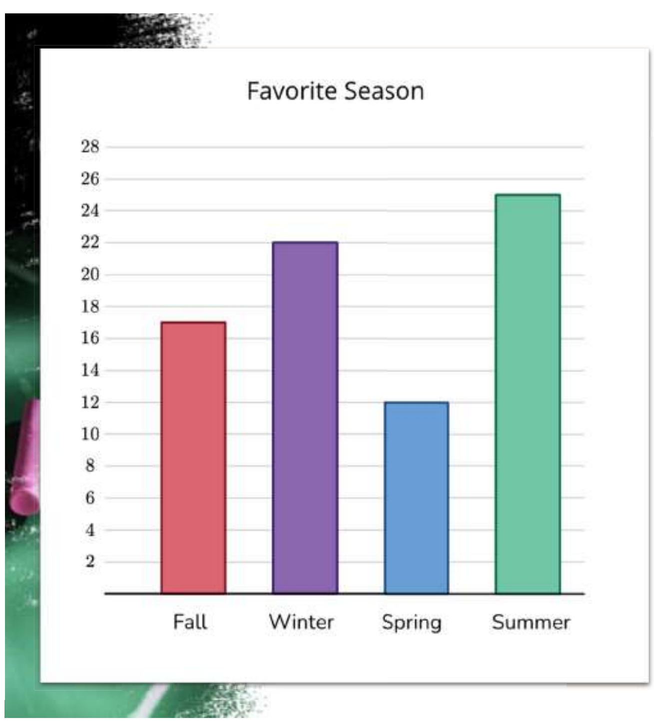
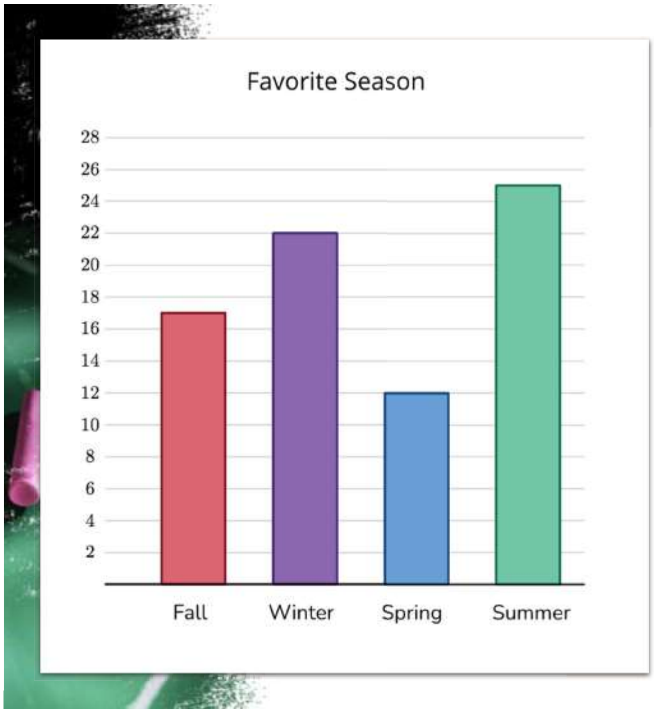


By *problem posing* in mathematics education, we refer to several related types of activities that entail or support teachers and students formulating (or reformulating) and expressing a problem or task based on a particular context (which we refer to as the *problem context* or *problem situation*). (Silver, 1994)

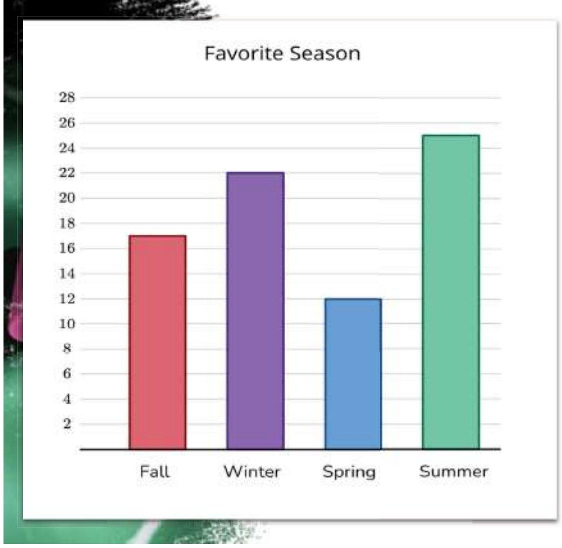


1. How many students are represented in the graph?
2. How many fewer students chose summer than fall and spring combined?



Pose one easy math problem and one challenging math problem that can be solved using this information.

How do the experiences compare?



Pose one easy math problem and one challenging math problem that can be solved using this information.

1. How many students are represented in the graph?
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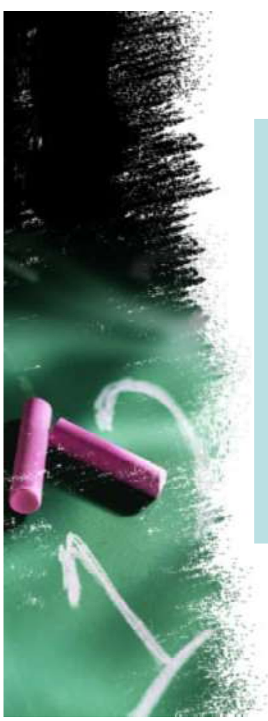
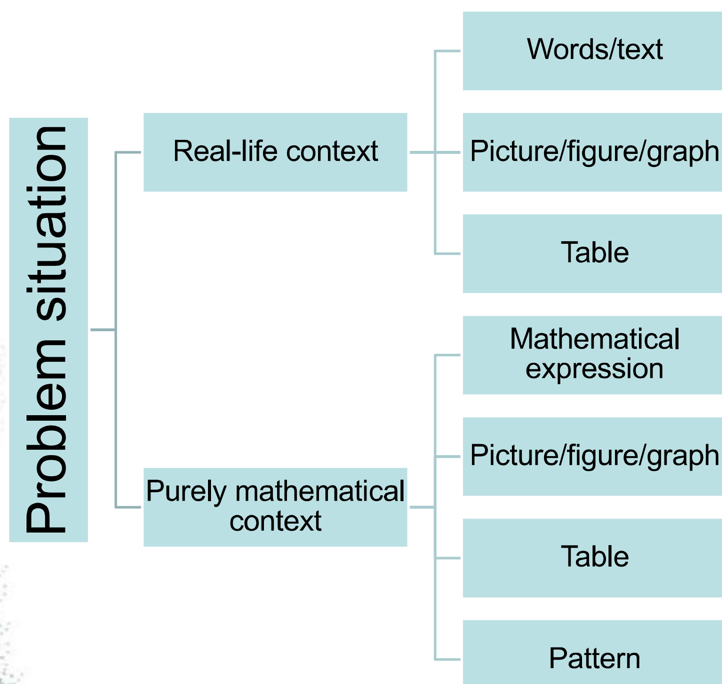
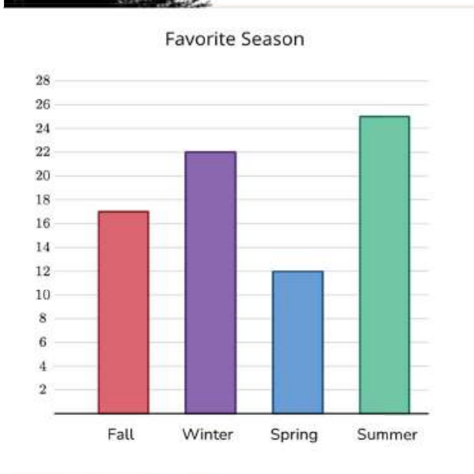
Problem-Posing Tasks

- Situations
- Prompts





Problem-posing Prompts



Prompt A: Pose three different mathematical problems that can be solved using this information.

Prompt B: Pose one easy mathematical problem, one moderately difficult mathematical problem, and one difficult mathematical problem that can be solved using this information.

Prompt C: Pose three mathematical problems to challenge your teachers to solve.

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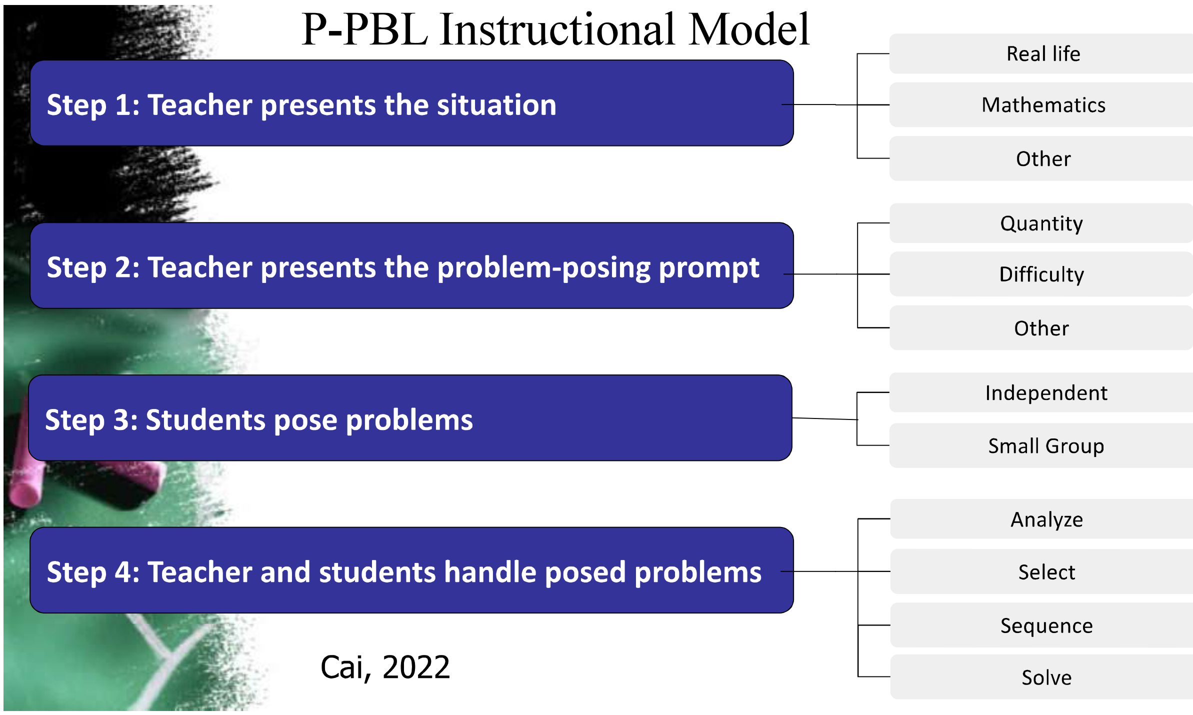
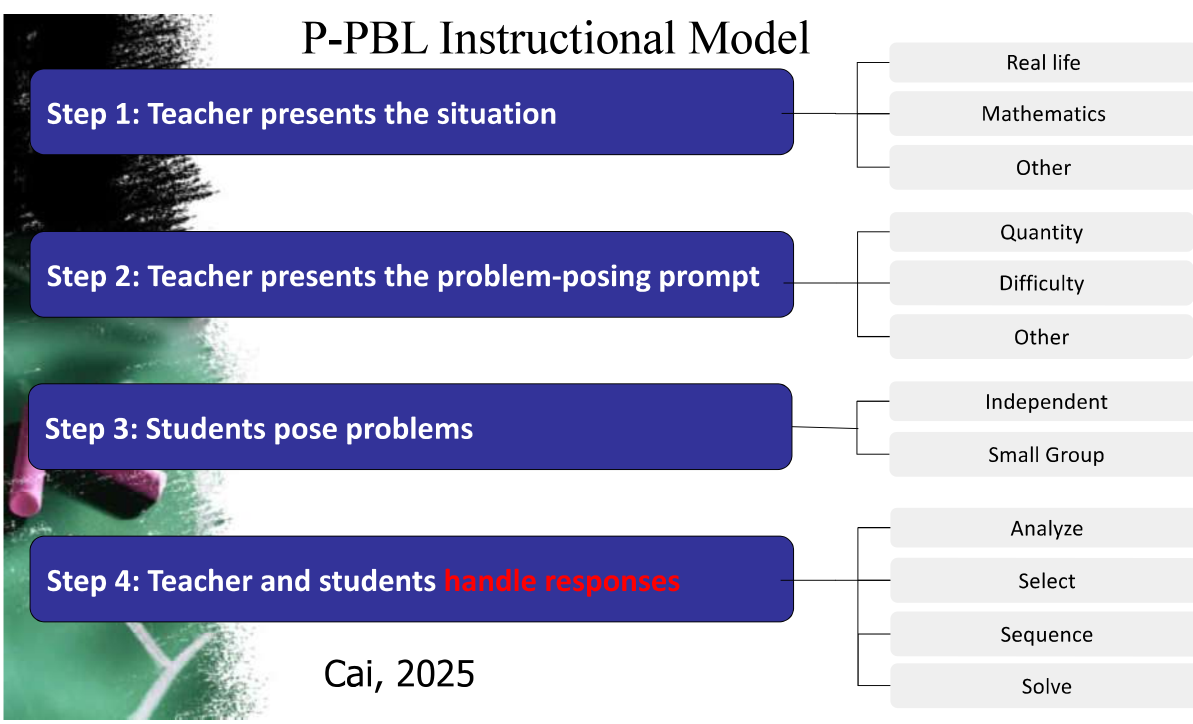

P-PBL Instructional Model
Step 1: Teacher presents the situation
Real life
Mathematics
Other
Step 2: Teacher presents the problem-posing prompt
Quantity
Difficulty
Other
Step 3: Students pose problems
Independent
Small Group
Step 4: Teacher and students handle posed problems
Analyze
Select
Sequence
Solve
Cai, 2022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Analyze
Select
Sequence
Solve
Cai, 2025

Selected Literature

1) English (1998)
2) Leung & Silver (1997)
3) Silber & Cai (2017)
4) Silber & Cai (2021)
5) Zhang et al. (2022)
6) Cai et al. 2023
7) Journal of Mathematical Behavior special issue in 2024

Leung & Silver (1997)

Situations

- With or without numerical information

Results:

The teachers performed better on tasks that included specific numerical information than on tasks without specific numerical information.

Further research – Task variables

a) How can systematic variation of PP situations and prompts inform our understanding of the relationship between PP processes and products as well as relationship between PP and PS?

b) How do student-related variables (e.g., knowledge, affect, experiences) interact with task-related PP variables?

c) How do problem-posing tasks (with variations) influence students' beliefs about mathematics?

d) How do problem-posing tasks (with variation) influence students' sense-making and motivation about mathematics?

- **Step 1: Teacher presents the situation**
  - Real life
  - Mathematics
  - Other
- **Step 2: Teacher presents the problem-posing prompt**
  - Quantity
  - Difficulty
  - Other
- **Step 3: Students pose problems**
  - Independent
  - Small Group
- **Step 4: Teacher and students handle responses**
  - Analyze
  - Select
  - Sequence
  - Solve

Models of the problem-*posing* process

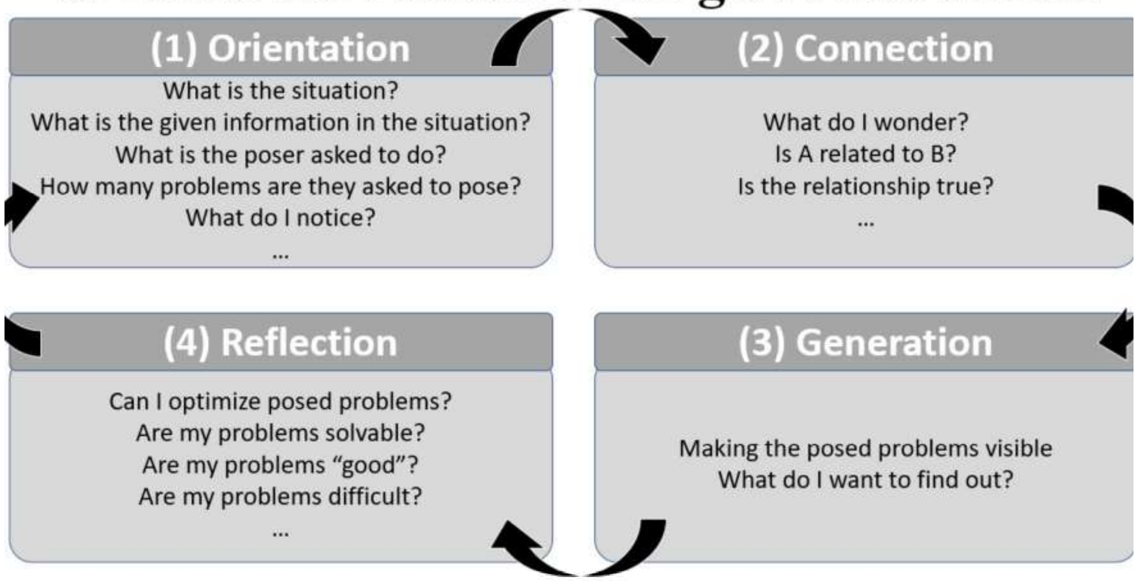
"[…] there is not yet a general problem-posing analogue to well-established general frameworks for problem solving such as Polya's (1957) four steps"

(Cai et al., 2015, p. 14).

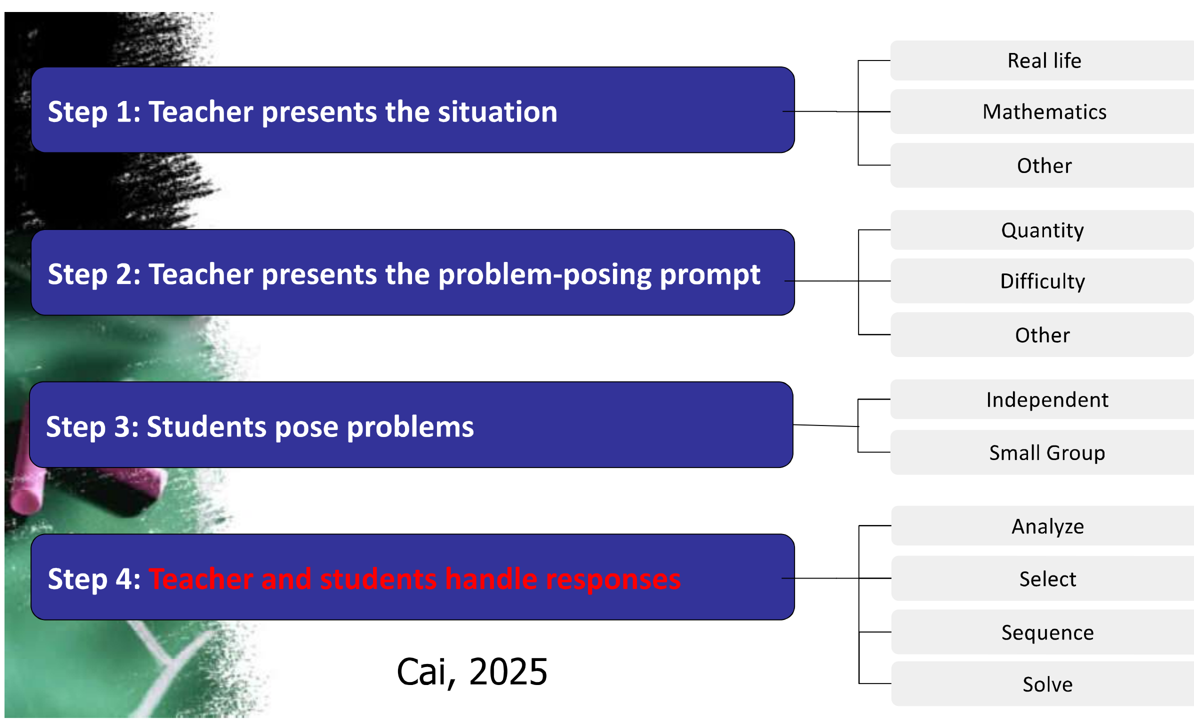
Problem-Posing Processes

- Processes through products (e.g., Silver&Cai, 1996)
- Reviewed models
  - *Cruz (2006)*
  - *Pelczer & Gamboa (2009)*
  - *Koichu & Kontorovich (2013)*
  - *Zhang et al. (2022)*
  - *Baumanns & Rott (2021, 2022)*
- Cai & Rott (2024)

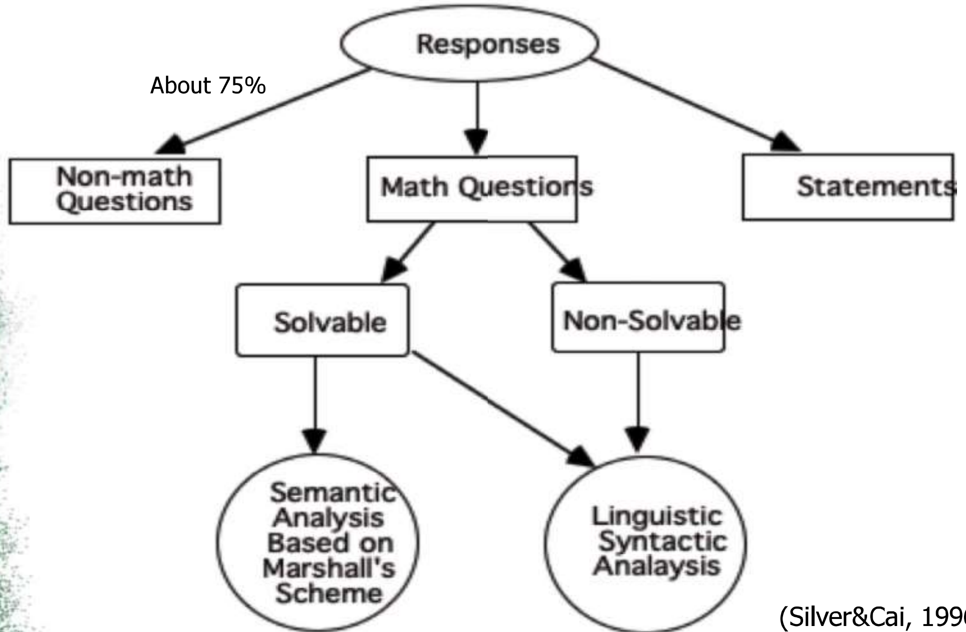
A General Problem-Posing Process Model



Cai & Rott, 2024



Cai, 2025



(Silver&Cai, 1996)

Outline

1. Why Problem Posing?
2. What does teaching mathematics through problem posing look like?
3. What is problem posing, anyway?
4. What is a problem-posing task?
5. How should teachers handle students' posed problems in classroom instruction?
6. What is the effect of P-PBL on teachers and students?
7. What is the next?

Meta-Analysis

- Ran, H., Cai, J., Hwang, S., Han, J., Ma, Y., & Muirhead, F. (in press). Effects of engaging in problem-posing interventions on learners' cognitive mathematics outcomes: a comprehensive meta-analysis. *Journal for Research in Mathematics Education*.

Outcomes	Hedges' g
Overall effect	0.53
Problem posing	0.84
Problem solving	0.52
Mathematics achievement	0.25
Affective outcome	0.67

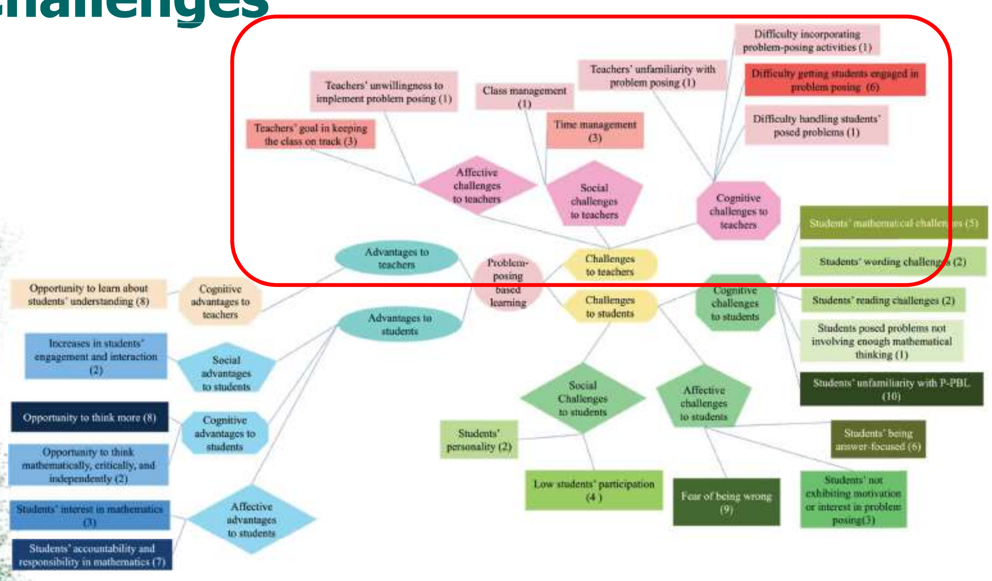
Measuring Changes

- Teachers' beliefs
- Teachers' lessons (improvement science)
- Their students' growth in cognitive and affective measures

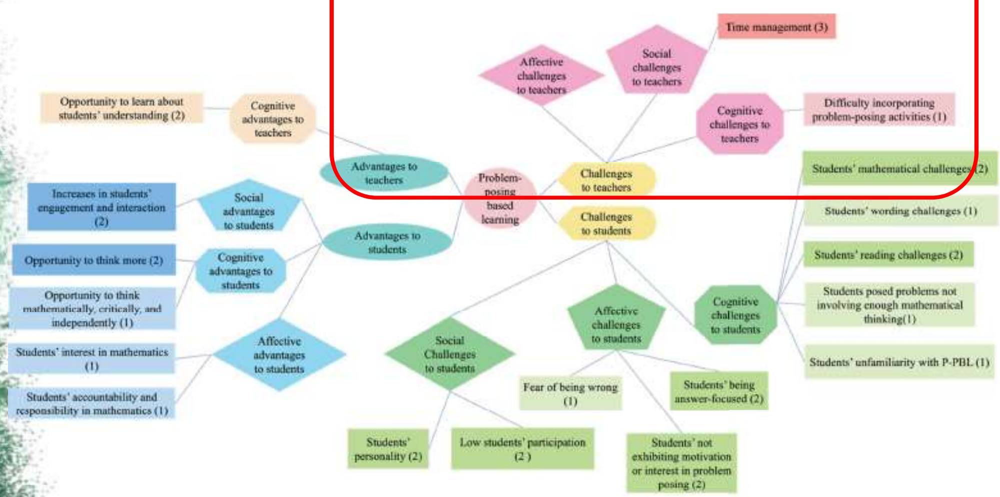
Teachers’ Perceived Advantages and Challenges

For teachers	For students
Cognitive	Cognitive
Affective	Affective
Social	Social

Teachers’ Perceived Advantages and Challenges



Teachers’ Perceived Advantages and Challenges



Mathematical Understanding and Problem Solving

- I was impressed … how they were engaged and what level of questions they came up with but after they were more focused on the content of the concepts of circles.
- [My students ask] more types of questions that involve a deeper level such as why and how. Questions that make them think or solve it before they can come to a conclusion or answer. I love how my students want to challenge themselves as well as others to come up with harder problems and not easy types of questions.
- I incorporated problem posing into my cool downs … by having my students think about mistakes that others could make on an exponent number line and create a new problem that might help students understand their misconceptions.

Student Engagement

- I noticed myself offering problem posing almost on a daily basis. This year I see students taking more responsibility in their own learning, and some have even told me that this year they have felt more comfortable in their math skills than in the past
- They especially like it when we use [problem posing] as an entrance activity and their problems/questions are extremely similar to ones that were in the textbook. This has really helped them connect to the material and engage more with the lesson.
- I noticed that shy students or students that have the mindset that “math is hard” were able to participate and show off their talents to me with enthusiasm, knowing that they may see their problem on an assignment.

Teacher Changes

- Feel much more comfortable with problem posing in classroom
- More lessons involving problem posing
- Using more problem-posing tasks

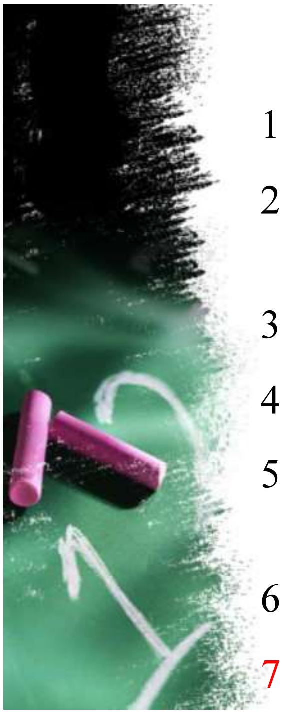
Problem-Posing Performance

- Growth in performance on problem-posing tasks
  - *Doorbell task: Each student poses three problems*

Sally is having a party. The first time the doorbell rings, 1 guest enters. The second time the doorbell rings, 3 guests enter. The third time the doorbell rings, 5 guests enter. The fourth time the doorbell rings, 7 guests enter. This pattern continues in the same way. On the next ring a group enters that has 2 more guests than the group that entered on the previous ring.

Problem-Posing Performance

- Compared scores of each student's best posed problem in the fall and their best posed problem in the spring
- For the Doorbell task (n = 282), the students' best posed problems in the spring tended to be scored more highly than their best posed problems in the fall (p = .002)



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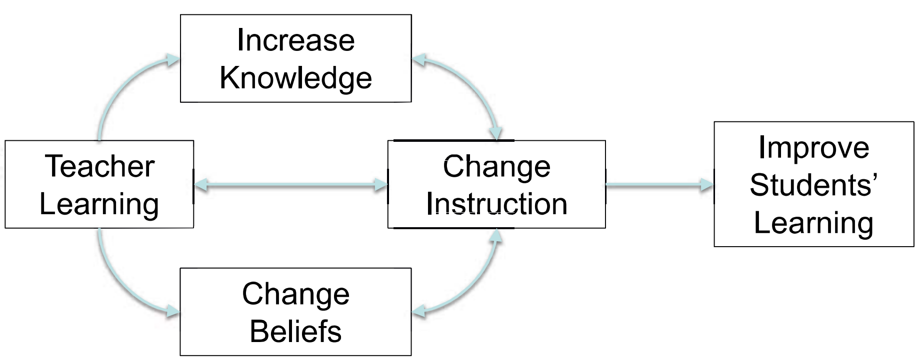
Investigating *longitudinally* how teachers learn to teach mathematics through Problem Posing and its impact on on classroom instruction and students' learning.



AI-enhanced problem posing, analysis of student posed problems, and P-PBL

A grant proposal under review by NSF

Thank you so much!

Jinfa Cai

jcai@udel.edu

Studying and Understanding Teaching Quality in Mathematics: Advancements Made and Challenges Faced

Charalambos Y. Charalambous

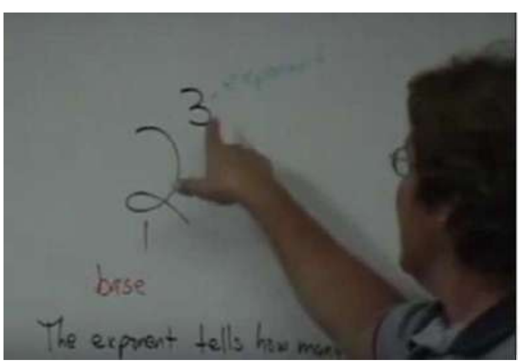
University of Cyprus
Department of Education

National Taichung University of Education, May 18, 2025
Innovation and Refined Teaching in Mathematics Education

Warm-Up Activity

2

How would you evaluate teaching quality in this lesson segment? Why?



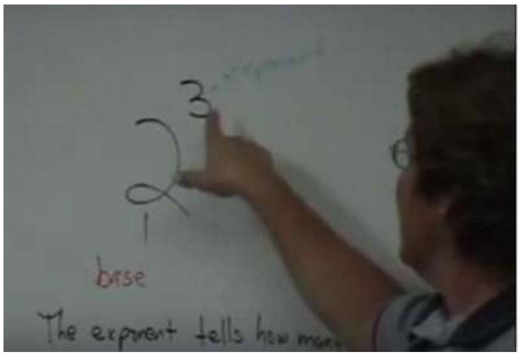
- 8^{th} grade mathematics lesson
- Focuses on calculating with exponents
- First lesson in a sequence of 12 lessons on this topic
- Lesson is 50 min in duration
- 36 students enrolled in the class

Source: TIMSS 1999 video study

Warm-Up Activity

3

Questions to consider:



- Do we all focus on the same teaching aspect(s)?
- On which teaching aspects should we focus? Does it matter?
- Even when focusing on the same aspect, do our evaluations converge?
- How do we know that we're producing valid and reliable estimates of teaching quality?
- …

Overview of the Presentation

4

- **Teaching quality: Starting from the basics**
  - Why focusing so much on teaching quality?
  - The construct of teaching quality
  - The role of classroom observations in capturing teaching quality
- **Advancements made and challenges faced**
  - Conceptualization
  - Operationalization
  - Measurement
- **Collaboration as a means of moving forward**

Why Focusing so Much on Teaching Quality?

5

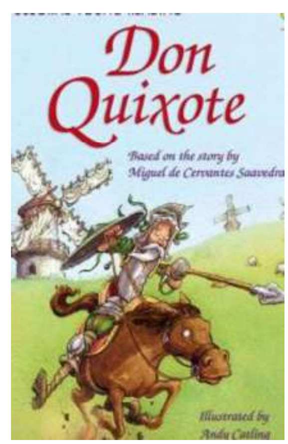
- Teachers are a key lever for student learning (Nilsen & Gustafsson, 2016; Nye et al., 2004; Rowan et al., 2002)
- Teacher effects have been found to explain a higher percentage of variance in student achievement compared to school-level and system-level effects (Kyriakides et al., 2020; Stronge, 2013; Reynolds et al., 2016)
- What matters most is *teaching* itself (what teachers ***do*** in the classroom with their students) rather than teacher attributes (Hattie, 2009, 2023)

Capturing Teaching Quality

6

"Quality in teaching is unusually difficult. Were anyone serious about this issue, they would soon realize that quality is an ineffable concept. […] Defining quality always requires value judgements about which disagreements abound."

(Berliner, 2005, p. 206)



Different Approaches to Capture and Examine Teaching Quality

7

Classroom observations

Student ratings

Teacher self-evaluations

Peer evaluations

Principal's evaluations

Teacher portfolios

Parents' evaluations

Value added models

On Classroom Observations

8

- Classroom observations often considered the gold standard for capturing teaching quality (Douglas, 2009; Rowan & Correnti, 2009)
- The field of classroom observation instruments clocks almost a century of life (cf. Charalambous et al., 2021; Martin, 1977)
- Classroom observations often carry a considerable weight in teacher appraisal and improvement systems worldwide (cf. Martinez et al., 2016)

Looking Back and Looking Forward

Advancements made

Challenges and Open Issues

Conceptualization
Operationalization
Measurement

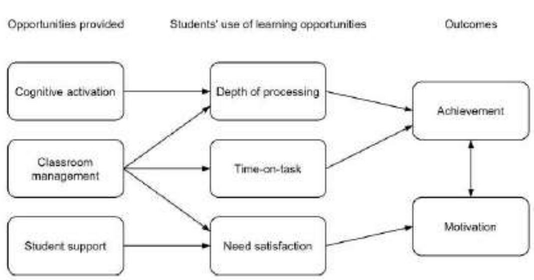
Indicative Questions

- **Conceptualization**
  - How is teaching quality conceptualized?
  - What teaching aspects are/can be observed?
  - How are the different teaching aspects observed organized/chunked?
- **Operationalization**
  - What is the focus of observation?
  - What is being captured when coding?
- **Measurement**
  - Are lessons parsed? If so, how?
  - What scales are being used?
  - At which level are the scores assigned?
  - What procedures are followed for rater recruitment, training, and certification? Are there any checks for rater drift?
  - Issues of validity and reliability in general

Looking Back at the Advancements of the Field

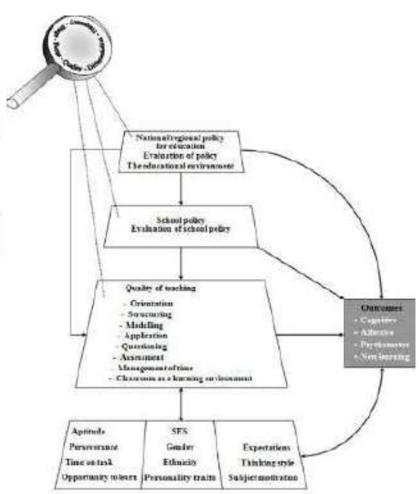
Conceptualization

- Moving from an a-theoretical field to the development of several frameworks (cf. Kyriakides et al., 2020; Scheerens, 2016)

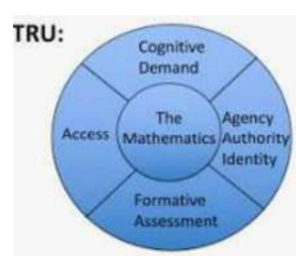


(Klieme et al., 2009)

(Pianta & Hamre, 2009)



(Creemers & Kyriakides, 2008)



(Schoenfeld, 2014)

Conceptualization

13

- **Adding subject-specificity into the picture**
  - Work on educational effectiveness research has for years focused on identifying **subject-generic** dimensions of teaching quality
  - Underlying assumption: effectiveness "travels" across content areas/subjects

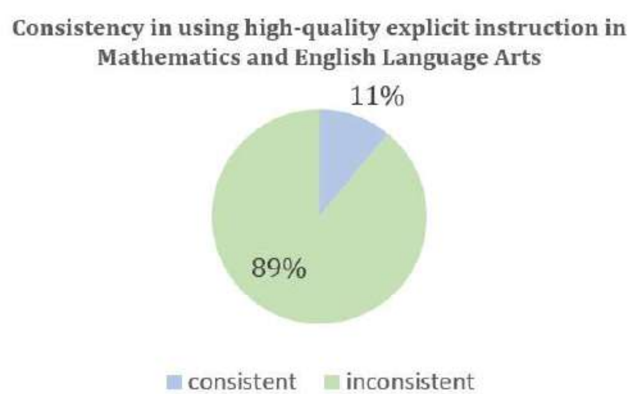
Conceptualization

14

- Research findings challenge this strong assumption!

	Low Effective in PE	Average Effective in PE	High Effective in PE
Low Effective in Mathematics	7, 8, 9, 10, 11	12	13
Average Effective in Mathematics	4, 14, 15	5	2, 3, 17, 18
High Effective in Mathematics	1, 16		6

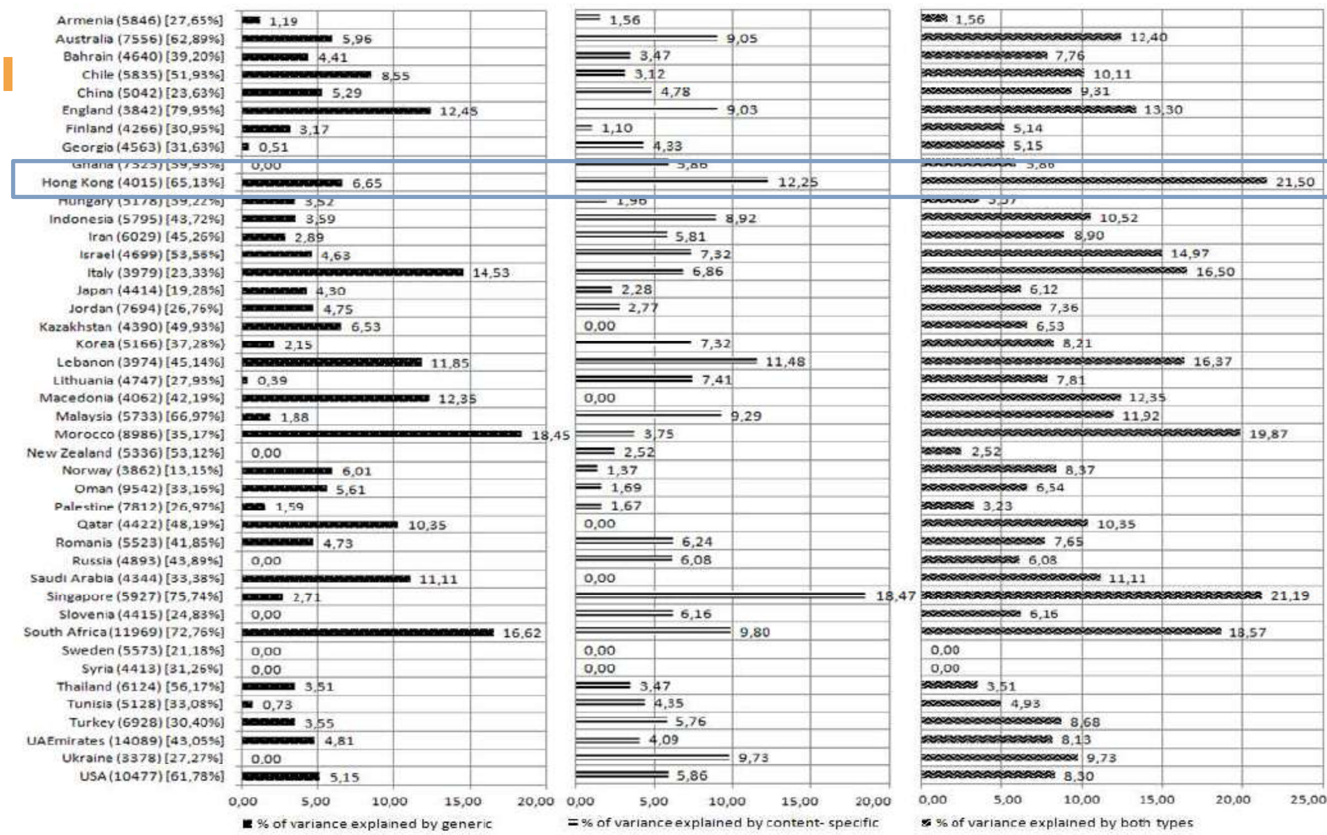
Clustered consistently: 39%
Clustered inconsistently: 61%



Charalambous et al. (2019)

Cohen (2018)

Conceptualization



Charalambous & Kyriakides (2017)

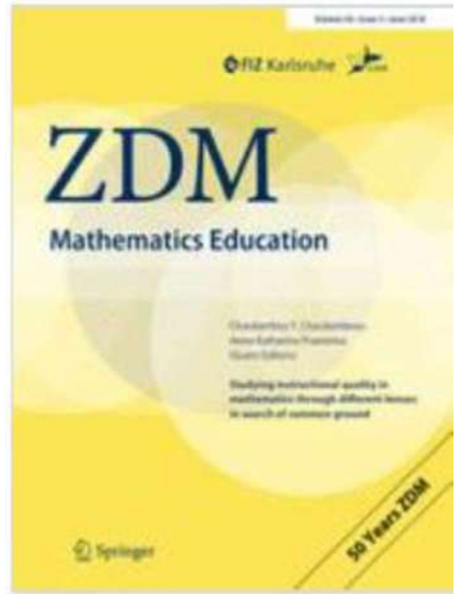
Conceptualization

- In general, attention to the role of the subject/content stressed even in the 1980s
- In educational effectiveness research discussion about the role of the content from the early 2000s (cf. differential effectiveness, see Campbell et al., 2003)
- But only more recently educational effectiveness researchers are attending to the role of the content more systematically
- ***Growing realization that content really matters***!

Reflecting on Work Done on Conceptualization, Operationalization, and Measurement

ZDM Vol. 50(3) Special Issue

- 11research groups representing 12 frameworks
- Groups analyzed the same three 4th-grade lessons using their own observational lenses
- Groups also provided information about issues of conceptualization, operationalization, and measurement



(Charalambous & Praetorius, 2018; Praetorius & Charalambous, 2018)

Conceptualization

Generic frameworks
- Developed and used to study teaching quality across different subject matters
- Attend to general demands of teaching

Hybrid frameworks

Subject-specific frameworks
- Developed and used to study teaching quality in a single subject matter (e.g., Mathematics)
- Informed by subject-specific demands of teaching within a particular discipline

Generic practices:
- Classroom management
- Time management
- Feedback providing (timely, relevant)
- Orientation
- Structuring

Subject-specific practices (in math):
- Precision and accuracy in mathematical language
- Linking different representations
- Generalizing and pattern noticing
- Contextualizing and decontextualizing

Conceptualization

19

Generic frameworks

- Developed and used to study teaching quality across different subject matters
- Attend to general demands of teaching

Hybrid frameworks

Subject-specific frameworks

- Developed and used to study teaching quality in a single subject matter (e.g., Mathematics)
- Informed by subject-specific demands of teaching within a particular discipline

Generic	Hybrid	Subject-specific
CLASS	MECORS & QoT	IQA
DMEE	TEDS-Instruct	MQI
ISTOF	TRU	M-Scan
TBD	UTOP	…
…	…	

Conceptualization

20

- Observational Foci can be Clustered into Seven Broad Categories:
  - Classroom and time management
  - Content Selection and Presentation
  - Cognitive Activation
  - Practicing
  - Formative Assessment
  - Socio-emotional support
  - Cutting-across aspects

Generic (G)/ Specific (S)	Teaching practices that create opportunities for student learning	Students' use of these opportunities (U) *	Content-generic frameworks				Mathematics-specific frameworks			Hybrid frameworks			
			CLASS	DMEE	ISTOF	TBD	IQA	MQI	M-Scan	TEDS-Instruct	TRU	UTOP	MECORS & QoT
	I. CLASSROOM AND TIME MANAGEMENT	Students are on task *(explicitly mentioned)*	U	U								U	U
G	Behavior management												
G[+S]	Time management												
	II. CONTENT SELECTION AND PRESENTATION	Students identify the reason(s) for engaging in certain activities; students use mathematics (language) accurately and precisely						U					
S	Selecting mathematically worthwhile and developmentally appropriate content												
G*S	Motivating the content							b					
G*S	Presenting the content in a structured way							b					
S	Presenting the content in mathematically accurate and correct ways												
	III. COGNITIVE ACTIVATION	Students are cognitively activated (e.g., explaining, justifying, generating ideas and conjectures, reasoning)	U			U	U	U	U	U	U	U	
G*S	Potential for cognitive activation through: (a) teacher's selection of challenging tasks which respond to students' cognitive level (b) teacher's use of mathematically rich practices												
G*S	Teacher facilitation of students' cognitive activity												
G*S	Teacher supports students' meta-cognitive learning from cognitively activating tasks												
	IV. PRACTICING	Students apply the procedures learned to known situations					U [c]		U [c]				
G*S	Teacher supports students solidify their procedural knowledge/skills												
G*S	Teacher procedural remediation of students' difficulties and errors in practicing												
	V. (FORMATIVE) ASSESSMENT	Students make use of the feedback provided to them									U		
S	Assessment is aligned with learning objectives												
G	Teacher regularly checks for understanding												
G*S	Quality of feedback for students												
G*S	Teacher capitalizes on formative assessment information to guide next instructional steps												
	VI. SOCIO-EMOTIONAL SUPPORT	Students feel socially related and emotionally supported											
G	Teacher-student relationships												
G	Student-student relationships												
	VII. CUTTING-ACROSS INSTRUCTIONAL ASPECTS AIMING TO MAXIMIZE STUDENT LEARNING	All students take ownership of their own learning and are productively engaged, according to their zone of proximal development	U		U		U	U			U		
G*S	Forming an environment that nurtures productive habits (e.g., agency, ownership/autonomous learning, identity, perseverance)												
G*S	Differentiation and adaptation												
G	Enhancing participation and the active engagement of all students												

Conceptualization

- ***Differences in conceptualizing the same teaching aspect***
  - ***For cognitive activation:***
  - Generic frameworks included indicators attending to engaging students in generating hypotheses, examining and analyzing data, and generating explanations
  - Math-specific frameworks provided more mathematics-sensitive manifestations of such behaviors, including posing mathematically motivating questions, offering examples and counter-examples, identifying patterns, and engaging students in mathematical reasoning

Conceptualization

23

- ***Differences in conceptualizing the same teaching aspect can lead to differences in what is observed and how it is evaluated***
- Using "quality of feedback" as an example:
  - "ISTOF: students' responses were corrected promptly and appropriately"
  - TEDS-Instruct: teacher "was very fast in correcting student errors without using mistakes or ideas as a learning opportunity"
  - TBD: the teacher "rarely provided constructive feedback"
  - CLASS: feedback was supportive for some students but not for others

A common lexicon of teaching is highly needed

Operationalization

24

- **Operationalization:**
  - the process of turning abstract concepts or ideas into observable and measurable phenomena (https://dovetail.com/research/operationalization)

		Number of Framework Elements		Observational Foci					
				Types				Key Observed Subjects	
		Level-1 elements	Level-2 elements	Frequency & Quality	Frequency	Quality	Other aspects	Teacher	Student(s)
Content-generic	CLASS	4	12	×				×	(×)
	DMEE	8	40		×	×	×	×	
	ISTOF	7	21			×		×	
	TBD	3	21		×	×		×	(×)
Content-specific	IQA	2	8	×				×	
	MQI	4	20	×				×	(×)
	M-Scan	9	22	×				×	
Hybrid	TEDS-Instr.	5	27	×				×	
	TRU	5	Variable	×		×			×
	UTOP	4	27	×				×	
	MECORS &	8	57		×			×	(×)
	QoT	9	26	×				×	

Operationalization

- To some extent, field seems like the Biblical Tower of Babel:
  - different names used to denote similar terms: domains, dimensions, components, indicators, items, codes, etc.
  - variation in the number of "dimensions/sub-dimensions" used
  - more emphasis placed on what the teacher does and less emphasis on whether and how students use the opportunities crafted for them to learn

→ More consistency needed in the lexicon we use to denote and capture the structure of teaching quality

→ Focusing more on how students **make use of the opportunities** crafted for their learning

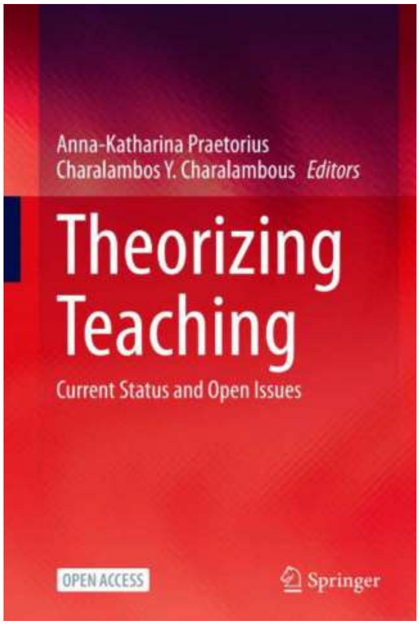
Measurement

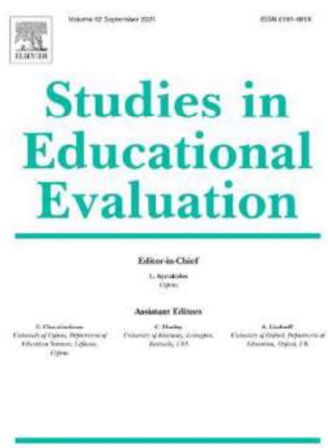
- Notable differences across frameworks:
  - Lesson parsing (time-wise or event-wise) vs. coding holistically
  - Scales used (4-point up to 7-point)
  - Assigning scores at different levels (Level-1 vs. Level-2 vs. Level-3)
  - Using multiple raters vs. using single raters
  - Quality assurance procedures used at various degrees
  - Reconciling scores vs. taking average scores
  - Different approaches used to examine validity and reliability

→ Need more consistency in the designs used and the rationales informing certain design decisions
→ More systematic and comprehensive work needed

Challenges and Open Issues: Looking Forward

Reflecting on Conceptualization, Operationalization, and Measurement Challenges





Merits and Limitations of Researching Teaching Quality More Synergistically (2020-2023)

Non-Convincing Effects in Research on Teaching Quality: Reasons and Possible Solutions (2025)

Conceptualization

- A plethora of classroom observation protocols currently being available (Bell et al., 2019; Bostic et al., 2021; Charalambous & Praetorius, 2018; Dobbelaer, 2019)
  - Fragmentation and local production of frameworks and protocols (Klette & Blikstad-Balas, 2018)
  - Only in mathematics education: 27 different observation protocols used in studies published in between 2000-2015 (Bostic et al,., 2021)

- Not clear which **theories of teaching** are guiding the production of these frameworks, let alone whether we have theories of teaching and what these really are (Praetorius & Charalambous, 2023)

Conceptualization

31

- **Could this proliferation of frameworks /protocols be potentially harmful?**
- If these frameworks and protocols come from different perceptions of teaching quality, this proliferation may suggest that "the field does not yet share a common vision of what counts as 'quality' mathematics instruction or what qualities should be emphasized when measuring instructional quality in mathematics" (Litke et al., 2021)
- Rapid production of frameworks and instruments could inhibit scientific development and progress (Charalambous & Praetorius, 2022)

Conceptualization

32

- **Can more synthesized models help bring some order?**
  - Working within each category to compare frameworks/models
  - Comparing what is learned from using the frameworks/models within each category to using a more synthesized model
    - MAIN-TEACH model (Charalambous & Praetorius, 2020)

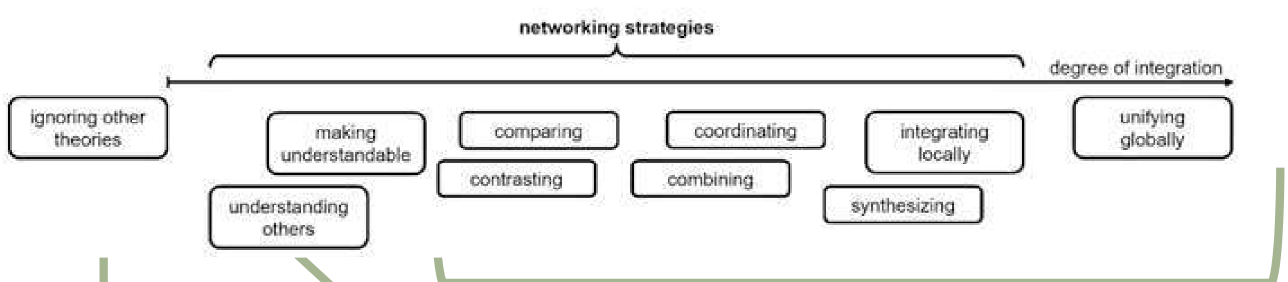
Conceptualization

- Why is this work necessary?

The mathematics-specific frameworks/protocols do not capture important aspects of instructional quality that are not mathematics-specific (Litke et al., 2021)

Conceptualization

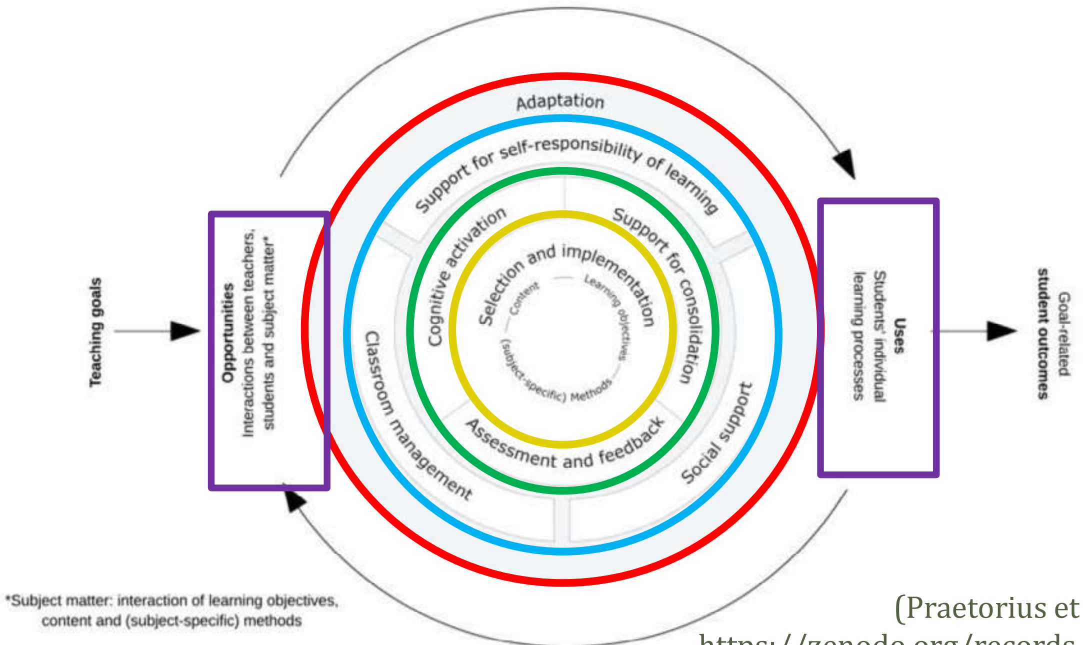
- Why is this work necessary?



(Bikner-Ahsbahs & Prediger, 2010)

Conceptualization

The MAIN-TEACH model (A multi-layered integrated model on teaching quality)**: An example of a synthesized framework**



(Praetorius et al., 2023)
https://zenodo.org/records/8280389

Conceptualization

Synthesized frameworks vs. Single Frameworks

Affordances	Limitations

(Charalambous & Praetorius, 2022; Lindorff et al., 2020; Litke et al., 2021; Panayiotou et al., 2021)

Conceptualization

37

(Charalambous et al., 2025)

Reflecting on Construct Decomposition:

- ✓ Decomposing teaching into broad dimensions and discrete elements within those
  - ✓ Helpful to take us beyond looking at isolated behaviors, but may occlude complexity of teaching and its quality
- ✓ Alternatively: considering how different dimensions of teaching interact or work in synergy (Hiebert & Stigler, 2023)
  - ✓ Example for cognitive activation and differentiation/adaptation (Charalambous, Agathangelou, et al., 2023)

Operationalization

38

- More systematic research that examines the impact that different operationalization decisions have for the outcomes obtained is needed:
  - number of elements included in an instrument and their organization
  - observational foci when trying to capture teaching quality (e.g., frequency, quality, stage)
  - attention foci (teacher and/or the students) and the need to capture how students make use of the opportunities provided to them

Measurement

- Empirical evidence highly needed for the impact that different measurement decisions have on the quality of the data obtained
  - Lesson parsing and lesson duration observed
    - Unit of measurement (from lesson segments to sequences of lessons)
    - Teachers' perspectives could also be informative (Ligozat & Almqvist, 2017)
  - Rater training (could disagreements be informative?)
  - Estimation of teaching-quality scores
    - Individual scores? Reconciled scores? Average scores?
    - Mean performance? Maximum performance?
  - …

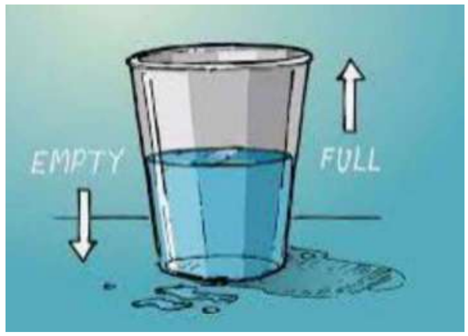
Additional Issues and Challenges

- How can observations work complementarily to other approaches of studying teaching quality?
- Adding issues of equity into the equation:
  - Whom are we observing and how? Is teaching supporting different groups of students? How?
- Using classroom observations from a more formative perspective
  - Providing teachers a space to learn and grow
- …

So, Where Are we Now?

41

The field seems to be a mess! Indeed, it is difficult to study teaching quality validly, reliably, and efficiently through teacher observations.



There are significant challenges and open issues—but that is important for continuing our research work!

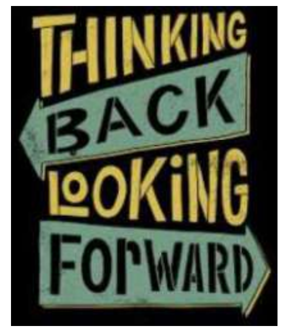
Looking the Glass as Half Full

42

- **Within challenge lies opportunity**
  - In Greek, the term "πρόκληση" ("proklisi", challenge) differs only a letter from the term "πρόσκληση" ("prosklisi", invitation to act)
  - In Chinese, the term "challenge" is composed of two symbols denoting "danger" and "opportunity"

危机

Collaboration as a Means of Addressing Challenges and Open Issues in the Field



Why Is Collaboration So Much Needed?

- Helps in comparing findings across studies and for more cumulative work (Grossman & McDonald, 2008; Praetorius & Charalambous, 2018)
  - A field cannot move forward if the research produced is not cumulative resulting in "a multitude of studies that cannot support inferences about the generalizability of findings nor sustain the long-term theory building that drives scientific progress" (National Research Council, 2005, p. 36)
- Necessary to build knowledge relevant for practice and research (Klette & Blikstad-Balas, 2018)

Working More Collaboratively on Classroom Observations: Some First Steps/Challenges

- Establishing common goals and agendas
- Resolving differences in terminology and structure
- Addressing operationalization and measurement challenges
- Dealing with insufficient transparency and the need for sharing information
- Attending to issues of limited funding

(Charalambous et al., 2021)

Thank you for your attention!

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Technology for teaching and learning mathematics: Opportunities and issues

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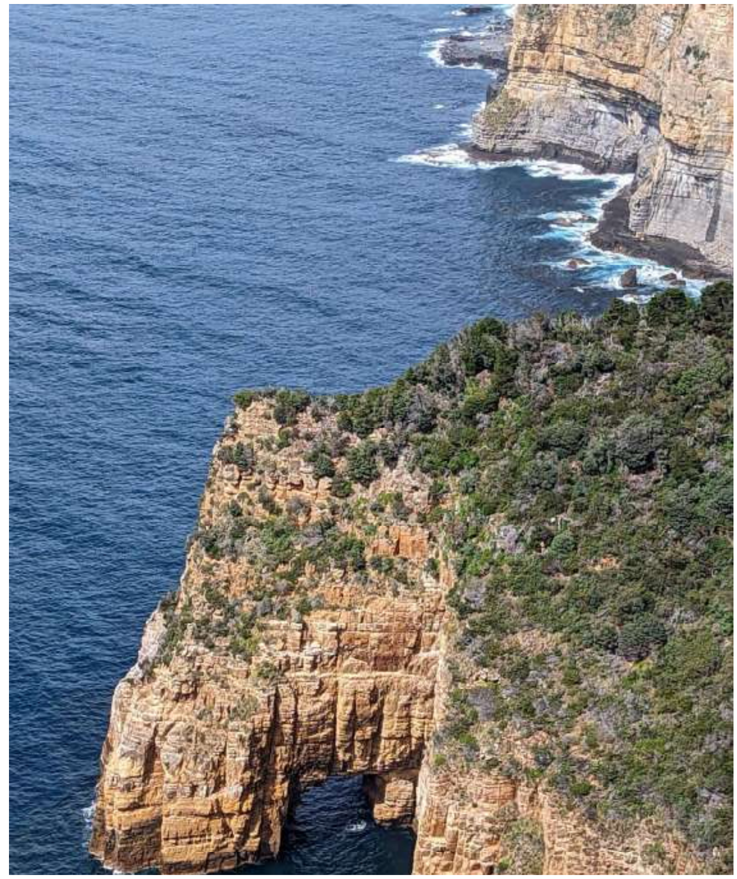
This presentation

- Curriculum imperatives for technology use
- Communication *through*, *with* and *of* technology
- Pedagogical opportunities with technology
- Principles for mathematics teacher professional development

- Examples presented focus on use of a Computer Algebra System

Curriculum and assessment imperatives for technology use

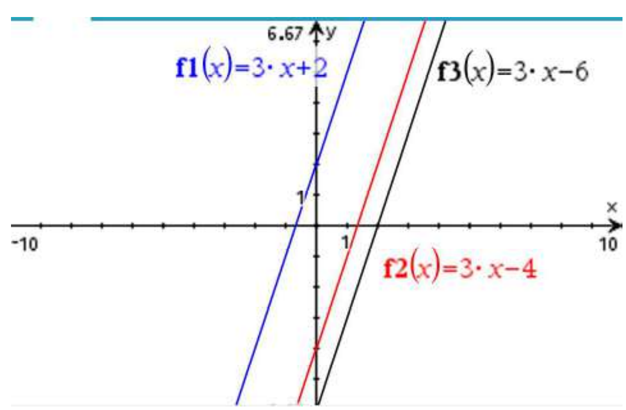


Curriculum

- In Victoria, Australia statements about technology have been explicitly included in curriculum documents for decades (e.g., Victorian Curriculum and Assessment Authority, 2022; n.d.)

- One Year 8 algebra curriculum statement

"experiment with linear functions and relations using digital tools, making and testing conjectures and generalising emerging patterns (VC2M8A05)"
(Victorian Curriculum and Assessment Authority, n.d.)



Assessment

In Victoria, Year 12 mathematics students expected to have a technology with computer algebra capability in an exam for each year 12 subject; there are also technology-free exams (Victorian Curriculum and Assessment Authority, 2022)

Where assessment 'expects technology', or is 'technology free' there are implications

- Nature of assessment
- Teaching
- Learning

Alignment of curriculum, pedagogy and assessment (Leigh-Lancaster & Stacey, 2022)

International interest in implementation of technology in mathematics education

- B. Pepin, Guedet, G., Choppin, J. (Eds.) (2024). *Handbook of Digital Resources in Mathematics Education, Springer International Handbooks of Education.* Springer. https://doi.org/10.1007/978-3-031-45667-1

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Communication *through*, *with* and *of* technology – Opportunities and issues for teaching and learning

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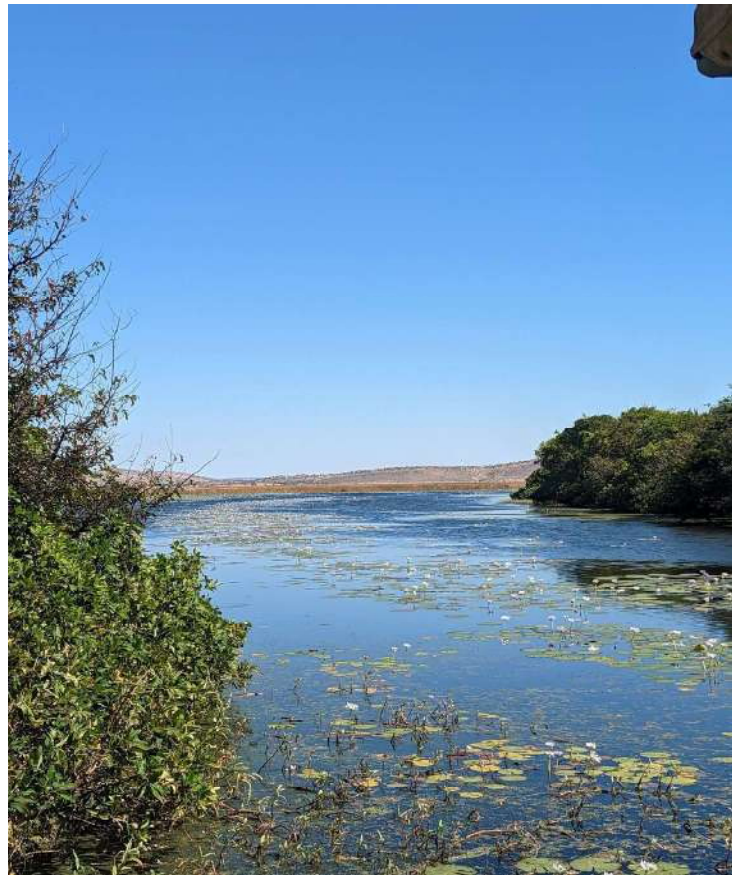


General-purpose and specific-purpose technologies

Development of mathematical knowledge

- **General-purpose technologies**

  Can be used across a range of lessons e.g., dynamic geometry, computer algebra system

- **Specific-purpose technologies**

  Might scaffold learning of one procedure or development of one concept e.g., applet

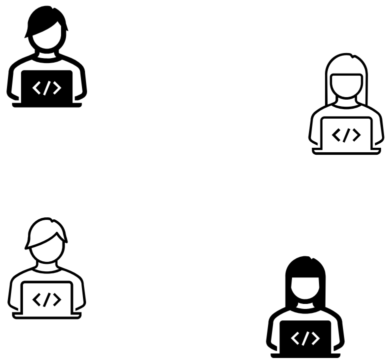
(Drijvers, Barzel, Mashchietto & Trouche, 2006)

- **Learning of syntax and ease of use of technology**
- **Time investment to learn to work with the technology**

Communication *through* technology

- E.g., Social networks to support students to work on problems collaboratively

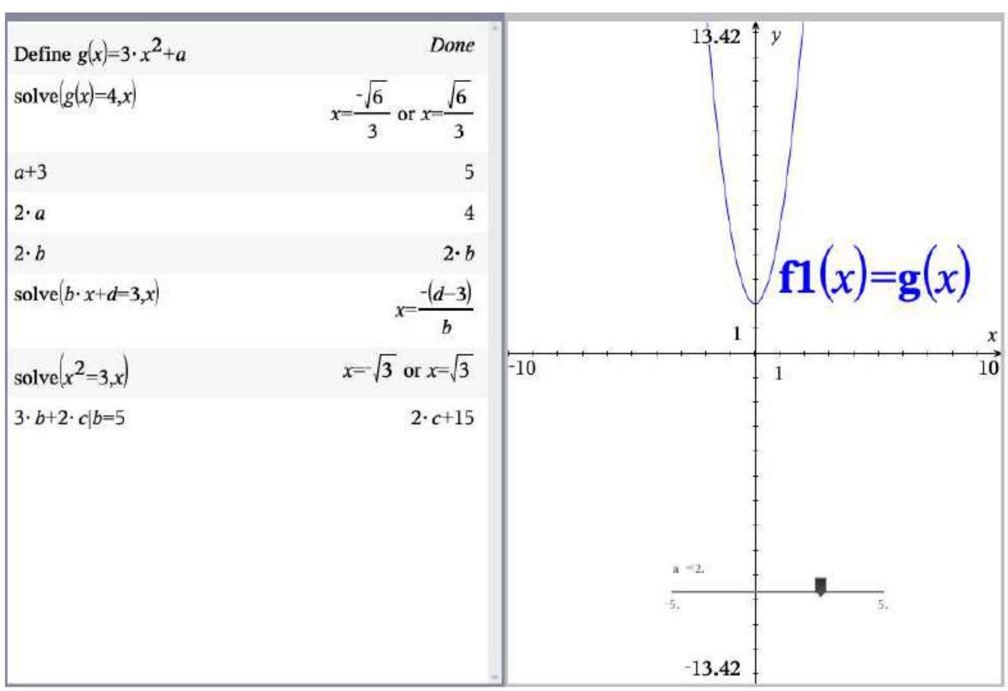
(Ball & Barzel, 2018)



Communication *with* technology

Communication that results in technology producing a display (e.g., syntax entry)

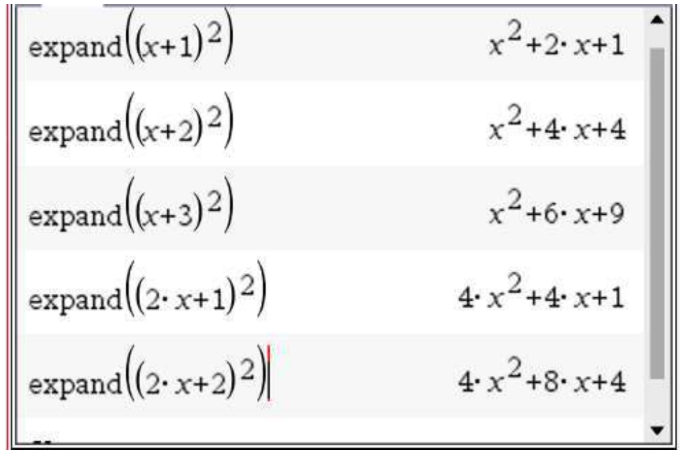
(Ball & Barzel, 2018)



Communication *of* technology

Technology display prompts communication

(Ball & Barzel, 2018)

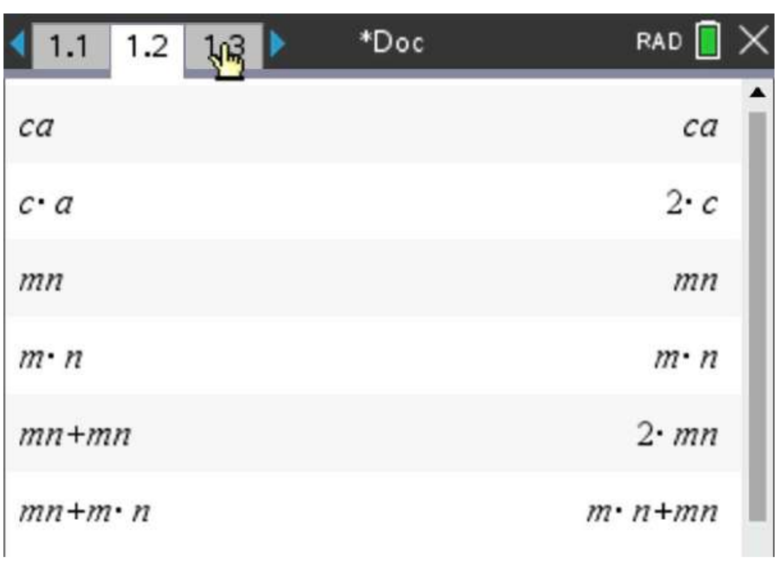


Communication *of* technology

Variable naming was an issue in setting up a multiple screen investigation (Pierce, et al., 2011)

Inability to have b and B as different variables.

Treatment of variables may be technology specific



'a' defined as '2' on page 1.1

No implicit multiplication between letters

One letter vs two letter variables

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Pedagogical opportunities

“Pedagogical map for mathematics analysis software”

Pedagogical opportunities

“Subject
(Re-assessed goals and methods

Classroom
(Improved display, personal authority)

Tasks
(Improved speed, access, accuracy)”

Functional opportunities support the pedagogical opportunities

Associated **curriculum** and **assessment** changes

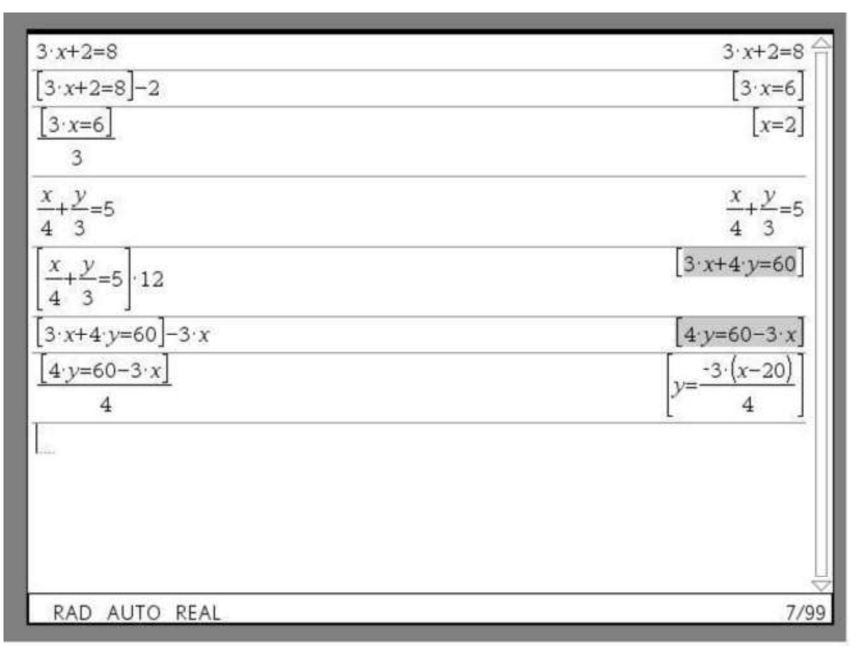
(Pierce & Stacey, 2010, p. 6)

Task level opportunities

Improved speed, access and accuracy:

Learn pen and paper skills

(Pierce & Stacey, 2010)



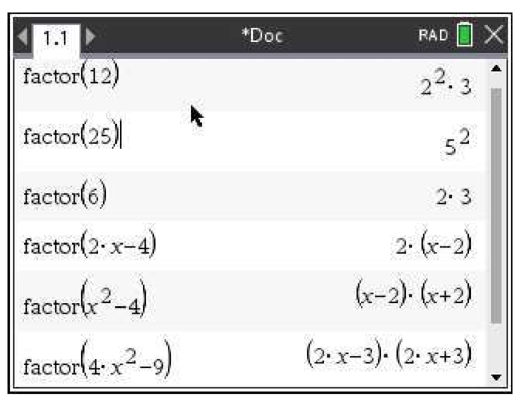
- Functional use of technology to carry out routine procedures
  - E.g. Simplify expressions (line 3 which gives x=2)
- Compensate for weak pen-and-paper skills
  - e.g. Line 5 where both sides of the equation are multiplied by 12

Subject level opportunities

Rebalance emphasis on skills, concepts, applications

(Pierce & Stacey, 2010, p. 6)

Using a CAS to factorise and then discuss reasoning for a pen-and-paper approach



Classroom level opportunities

Classroom level opportunities

Didactic contract can change

- Technology can be an additional ‘expert’ in the classroom

Göbel, Barzel & Ball (2017) report a study where three groups of students used different types of visualisation of functions supported by technology. The students were involved in discovery learning, so the teacher was a facilitator, rather than directing each lesson. Technology was found to support students’ understanding of the role of parameters in functions.

Technology assisted guided discovery

Four Year 9 groups – control, ”function plotter”, ”drag mode”, “sliders”
Explore and explain role of parameters in $f(x)=a(x-b)^2+c$ and produce a summary sheet to summarise learning

Findings

- Supported learning of concept of parameter, particularly through “dynamic” and “sliders” (a=0 case noted)
- Supported exploration, with some students choosing own examples
- Supported hypothesis testing

(Göbel, Barzel, & Ball, 2017)

Stability of students’ beliefs about use of technology

- Study of 12 senior secondary mathematics students’ beliefs about technology use in mathematics
- Studied a mathematics subject (included calculus and algebra) where use of technology was expected
- Technology used was a Computer Algebra System (CAS)
- Research instrument – Questionnaire with one section related to beliefs, administered April and November
- Items based on literature review (further information in the article)

Selected student beliefs about CAS related to

- Usefulness
- Speed compared to pen-and-paper (p&p)
- Ease of use
- Whether using CAS is ‘proper mathematics’
- Correctness of answers
- Choice of CAS or p&p

(Cameron, Ball & Steinle, 2023)

Students' beliefs

'Very stable' beliefs

- CAS is useful for solving problems quickly
  (held, n=10)
- P&P needs to be used before CAS
  (held, n=10)
- There is a need to choose between CAS or P&P before solving a problem
  (held, n=2; not held, n=8)

'Inconclusive' beliefs

- Most algebra problems can be solved faster with CAS than P&P
- It is easier to solve problems with CAS than P&P
- It is hard to learn how to solve problems with CAS
- Mathematics is not being done properly if CAS is used

(Cameron, Ball & Steinle, 2023)

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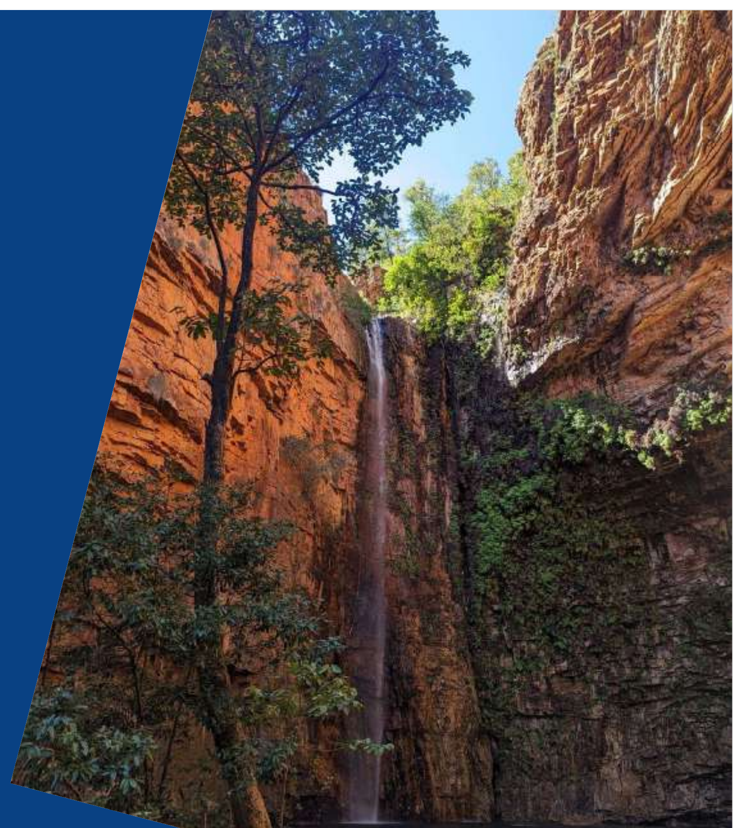
Principles for mathematics teacher professional development



CERICS: Principles for professional development for teaching mathematics with technology

	Focus	Principle
*	Creativity	Promote teachers' creativity in the design of lessons and /or assessments that balance technology, pen-and-paper and mental strategies
*	Explicit PCK	Make pedagogical content knowledge for teaching mathematics with technology explicit
*	Reflection	Plan focused reflection by teachers on the role and impact of technology
	Investigate student thinking	Include discussions of classroom scenarios and/or student work samples to highlight affordances and constraints of technology
*	Collaboration	Plan for interaction and exchange between teacher participants to share pedagogical approaches and experiences of teaching with technology
	Self-efficacy	Reduce teachers' anxiety about using technology and promote their risk taking in use of technology

(Ball & Barzel, 2024, p. 36)

CREATIVITY Promote teachers' creativity in the design of lessons and /or assessments that balance technology, pen-and-paper and mental strategies

Teachers

- Decide on the balance between technology, pen-and-paper and mental strategies
- Identify general principles for teaching from examples
- Create specific examples from general principles

(Ball & Barzel, 2024, p. 36)

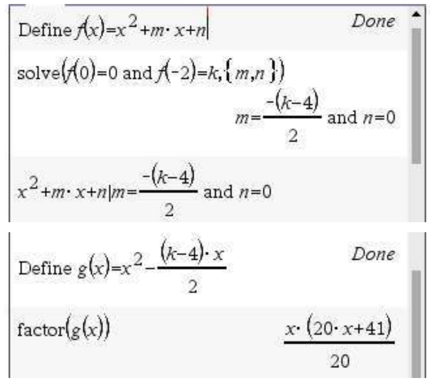
Opportunities for students' learning
Students experience targeted examples that highlight the efficacy of technology, p&p or mental strategies
Strategies for using technology, p&p and mental strategies are included in classroom discussion

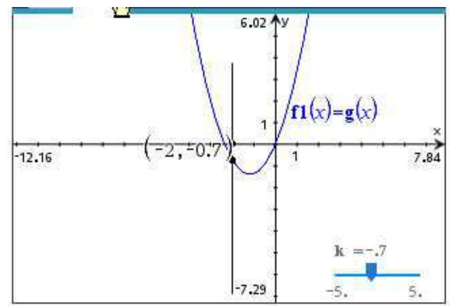
CREATIVITY: Teachers create a task from curriculum document description

(Ball & Barzel, 2024, p. 36)

"A practical or theoretical situation in which a curve passing through points on the plane is modelled by the graph of a polynomial function of low degree. This could involve exploring what information can be used to determine the equation of a linear, quadratic or cubic function, and what graphs can be found to fit particular sets of information. Key features of families of graphs that fit a particular set of information could also be explored. For example, a family of quadratic functions with a leading coefficient equal to 1, whose graph contain the points with coordinates $(0, 0)$ and $(-2, k)$. By systematically varying k, the effect of k on the behaviour of the graph of the quadratic function could be explored."

(VCAA, 2022, p.46)





Explicit PCK: Make pedagogical content knowledge for teaching mathematics with technology explicit

(Ball & Barzel, 2024, p. 36)

Reflection: Plan focused reflection by teachers on the role and impact of technology

Technology displays

Pedagogical issues (Limitations of technology, syntactical requirements, expect unexpected displays, pedagogical reflection)

Include technology tasks that result in lesson "hiccups" (Clark-Wilson & Noss, 2015)

Applicable for students who are learning mathematics with technology

(Ball & Barzel, 2024, p. 36)

Collaboration: Plan for interaction and exchange between teacher participants to share pedagogical approaches and experiences of teaching with technology

Collaboration also important for students who are learning mathematics with technology

Task design and sharing can support teacher learning (Ratnayake, et al., 2020)

(Ball & Barzel, 2024, p. 36)

Conclusion - Foster links between research and practice

Harness the potential of technology to improve outcomes for students

- Build mathematical knowledge, critical users of technology and positive disposition towards technology as a viable option in mathematics

Building teacher expertise in making pedagogical use of technology

- Teachers aware of implications of research and active participants in research
- Develop teachers' expertise in being discerning users of technology
- Teacher willing to take a facilitator role to support exploration with technology
- Further research on both general-purpose and specific-purpose technologies in a range of contexts

- Important consideration is to recognize underlying principles related to technology use in specific classroom implementations
- Consideration of technology in one context can add to the broader knowledge of technology use in teaching and learning of mathematics

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Thank you

2025 年第十七屆
科技與數學教育國際學術研討會
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5/17 口頭發表

時間	場次	編號	作者	論文題目
13:40-15:20	一	1	萬依萍、賴郁儒、李湘	AI 語言處理與情緒運算在數學學習行為測量上之應用
		2	賴彥廷、郭珈忻、秦爾聰	偏鄉教師專業學習社群融入 JDM 教材之行動研究
		3	曾子軒、劉柏宏、蘇柏鑫	數學史融入國小教學-九章算術圓田
15:40-17:20	二	1	陳宥諭	國中數學教科書試題之比較與分析－以二元一次聯立方程式為例
		2	趙子渝、林碧珍	臺灣與中國國小數學教科書在分數不同意義的數學活動類型之比較
		3	郭品辰、楊凱琳	臺灣高中數學教師對導數概念的認識論觀點
	三	1	吳岱穎、許慧玉	國小五年級視知覺及操弄之個案研究
		2	范喻婷、許慧玉	國小六年級擬題的數學創造力表現之研究
		3	李欣蓉、陳建誠	結合提問和數學感提升學生文字題解題表現之個案研究
	四	1	施柏帆、楊凱琳	八年級學生經歷數位數學遊戲學習等差數列的感受
		2	蕭籽芸、謝佳叡	數學教育相關研究結果在數學課堂實踐情況的調查研究：以差異化教學為例
		3	許祖綺	數學奠基模組融入國小六年級圓與扇形面積教學之個案研究
	五	1	石雅竹	六年級個案學生在比與比值及其相關單元的解題信念
		2	吳宜家、謝闓如	國小六年級學生錯誤類型之研究—縮圖、放大圖與比例尺為例
		3	劉家年、陳建誠	提升四年級學習扶助學生文字題解題能力之個案研究

2025 第 17 屆科技與數學教育國際學術研討會口頭發表場次

5/18 口頭發表

時間	場次	編號	作者	論文題目
13:30-14:30	六	1	林佳儀、蘇柏鑫	運用 Polya 解題策略教學降低五年級學生數學學習焦慮之行動研究-以臺中市某國小為例
		2	曾品瑄、蘇柏鑫	運用數學寫作於國小五年級整數四則計算之行動研究
		3	張榆平、林原宏	國小高年級學生數學情緒初探
	七	1	潘珮瑜	數學奠基模組融入國小四年級「假分數和帶分數互換」 教學之個案研究
		2	蔡鎧丞	以探究教學提升國小五年級學生數學推理與系統性思考之研究
		3	黃暉娟、吳慧敏	以《圖騰島》桌遊探討小學生空間推理能力之促進效果
	八	1	邱子倫、李源順	診斷教學應用於數學分數單元教學成效之研究
		2	魏愷呈、姚如芬	異分母分數加減的迷思概念探究與教學設計
		3	許悅蓉、林原宏	應用四階段評量診斷六年級學童在分數除法問題的表現
	九	1	林和傑	學生影片製作與討論促進數學參與：比較 YouTuber 型式與教學型式任務設計
		2	蘇柏昇、沈明勳	因倍數概念之學習診斷工具開發之前導研究
		3	林沁儀、林原宏	應用三階段評量探討國小五年級學童在因倍數問題的解題表現
14:40-15:40	十	1	洪詠馨、陳建誠	差異化教學中的彈性分組策略對國小六年級學生數學學習成效之影響
		2	何曉彬、楊凱琳	探究教師在數學數位差異化教學中轉變之理論框架
	十一	1	張娣安、李源順	國小二年級低成就學童數學外加課程舉例能力之教學初探
		2	林珅兆、姚如芬	國中數學課後學習扶助使用數位學習平台之個案研究
		3	許家穎、魏士軒	學生小組成就區分合作學習融入六年級小數與分數的計算教學之研究
	十二	1	史翊君、李源順	以數學感「舉例」策略融入一年級數的單元之初探
		2	林昀暄、林原宏	IMPROVE 後設認知教學法對國小學生數學學習影響初探

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二/謝佳叡教授
三/李源順教授
四/張育萍教授
五/魏士軒教授

AI 語言處理與情緒運算在數學學習行為測量上之應用

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摘要

本研究將建議使用量化方式，用於評估與測量小學低年級生學習數學之解題能力，並整合先進之方法來探討如何利用情感偵測學童數學解題之困境。本研究透過行為影像從顏面識別及聲紋探測學童之學習認知與情緒感知，透過情感運算系統能進一步透過語言分析，顯現學童數學學習成果之狀態與潛力。從顏面識別影像能提取認知行為特徵，藉此進一步評估學童解題所遇到之困難。透過聲紋偵測與多模態之間的相互作用，本研究期望提供進一步情感運算之的演算與模型，為更深入理解數學語境下學童溝通之情感層面與基礎。

關鍵字：情感運算、數學教育、聲紋語音、多模態

偏鄉教師專業學習社群融入 JDM 教材之行動研究

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摘要

本研究主要探討偏鄉地區教師如何透過教師專業學習社群（Professional Learning Community, PLC）融入 JDM 教材來提升數學教學知識為主軸，針對不同專長領域的偏鄉國小教師在教學現場如何提升自我的數學教學知識。
本研究使用的研究工具包含陳彥廷（2014）所提出的國小教師數學教學知識（MPCK）知覺量表來當作前後測、教學活動錄影、晤談語料與社群活動相關文件，並使用數學領域中央輔導團的「就是要學好數學(JDM)」教材來進行資料蒐集，對臺中市某偏鄉學校由四位老師組成的教師專業學習社群進行研究。

本研究的發現如下：一、JDM 教材結構清晰，對數學背景教師支持性強，但非數學背景教師適應過程中面臨挑戰；二、JDM 教材評量工具提升學生參與度，但教師期待更多分層設計與互動資源以支持多樣化教學；三、教師使用 JDM 教材後，在數學教學知識總體表現有所提升。

關鍵字：教師專業學習社群、JDM 教材、數學教學知識

數學史融入國小教學-九章算術圓田

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摘要

本研究為前導研究，旨在藉由擷取數學史相關書籍中之適當數學史素材進行教案設計，並進行實際教學以觀察學生思考行為和分析數學史教案對於發展學生數學問題解決的思維有何效益，以做為未來正式研究之參考。本研究之數學史相關素材多擷取自《幾何原本》、《九章算術》二書，其餘少部分則擷取自《數學起源》、《孫子算經》、《數之軌跡》等等數學史相關書籍。本研究所設計之教案採實務取向，並參考密西根大學教學研究中心(Center for Research on Teaching and Learning)所揭示的教案設計準則進行設計。於教案設計後採用本研究所發展之教案進行實際教學測試，結果顯示學生於課堂中展現高度的專注，並且也透過古人的解題方法與現代解題方法的比對後更了解課堂中所教授的主題內容。

關鍵字：數學史融入教學、數學史、數學文化、數學思維

國中數學教科書試題之比較與分析－以二元一次聯立方程式為例

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摘要

本研究採內容分析法，探討臺灣三家出版社「A 版」、「B版」和「C版」之國中數學教科書二元一次聯立方程式教材中「試題分布分析比較」、「題目認知類型分析」與「題目表徵型態」。研究發現三個版本各有特色與著重方向：A版本應用問題最多，C版最少；而解聯立計算B版本最多，C版最少。題目認知類型方面 A版本偏重無連結型題型，強調基礎計算能力；B版本偏重具連結型題型，著重理解與概念應用；C版本則在四種認知類型上分布均衡，強調學生循序漸進學習。題目表徵方面三版本數學型態題型佔比超過 75%，文字型態約佔 25%，顯示應用題的重要性。三版本素養題型均約佔 25%，集中於應用題，反映三版本對學生實際應用與解決問題能力的重視。

關鍵字：比較研究、數學教科書、教科書試題分析

臺灣與中國國小數學教科書在分數不同意義的數學活動類型之比較

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摘要

本研究目的旨在探討比較臺灣與中國小學數學教科書對於分數不同意義的編排。研究對象為臺灣 A 版本和中國 B 版本數學教科書。本研究採內容分析法，以數學問題為分析單位。研究結果發現:臺灣分數教材分布於八冊，從二上至六下，中國分布於三上、五下及六下共三冊。兩國教科書都是從部分-整體關係建立分數的基本概念，但是兩國教科書對於分數是部分-整體關係的詮釋不同，臺灣和中國對於分數六種不同意義的引入，各具有其特殊目的，臺灣 A 版本引入單位分量累加是為了建立假分數的概念，而中國 B 版本是為了處理真分數的同分母分數相加減。分數是兩數相除的結果的引入，臺灣 A 版本和中國 B 版本都是為了將分數化為小數，但是中國 B 版本將假分數化為帶分數是另一個目的。兩國教材皆以單位分量的累加為最高，部分-整體次之。臺灣和中國教科書對於分數意義的學習以透過「操作與計算」的活動類型最多，主要是學習「分數是單位分量的累加」，而透過「表示」活動主要是在建立分數的「部分-整體關係」意義，但是兩國的教科書都提供不到一成的「解釋」、「論證與推理」高認知需求活動類型給學生學習分數的六種意義。

關鍵字：分數意義、國小數學教科書、臺灣與中國

臺灣高中數學教師對導數概念的認識論觀點

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摘要

教師的認識論觀點（epistemological views）被視為是影響其課堂教學實踐的重要因素。而在高中微積分課程中，導數概念不僅是分析真實問題的重要數學工具，亦作為學習進階數學概念的知識基礎。有鑑於臺灣中學數學教師在國際 TEDS-M 研究中於數學內容知識（mathematical content knowledge）與數學教學內容知識（mathematical pedagogical content knowledge）方面均有優異表現，本研究採用現象學方法（phenomenographic approach），探討具豐富教學經驗之臺灣數學教師對導數概念的認識論觀點，並進一步提供對微積分學習與教學的啟示。

透過半結構式訪談，我們訪談六位在微積分學習與教學方面具高度自我效能感的資深中學教師，並運用主題分析法（thematic analysis）歸納出六大核心主題。在導數概念的本質與重要性方面，教師的觀點包括：(a) 過程觀與物件觀的導數理解方式，(b) 變化率與極限作為導數概念的關鍵基礎，以及 (c) 具有雙重視角及多維視角的導數概念認知。在導數概念的學習方面，教師的回應則呈現出不同觀點，包括：(d) 先備知識與學習技能的需求，(e) 對數學嚴謹性的要求，以及 (f) 透過多種例證建構不同概念間的連結。

本研究有助於深化對具高自我效能之有經驗教師如何理解導數概念本質與重要性的認識，並闡明其不同的理解方式。在理論層面，本研究所歸納的主題可進一步闡釋導數概念的內涵及其教學應用，並透過多樣例證促進概念間的連結。在實務層面，本研究結果可作為師資培育的參考，以回應與教師認識論觀點發展相關的教學議題。

關鍵字:臺灣高中數學教師、認識論觀點、微積分、導數概念

國小五年級視知覺及操弄之個案研究

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摘要

本研究旨在探討國小五年級學童在非正式的數學學習環境中，對於幾何圖形的視知覺情形為何，並探討學生會有哪些思考的過程。研究者參照 M. Kus & E. Cakiroglu(2022)中的研究方式，並稍作修改，只取簡圖部分訪談學生。本研究採用質性研究法中之個案研究法，主要採用半結構式訪談方式，對學生進行一對一訪談，研究者不會事先進行教學，施測時，受訪學生不受任何限制，只需依照訪談大綱自由的回答，研究者會依照學生回答的情形，適時追問或是給予指示，以深入探討學生在非正式的數學學習環境中，能引學生的視覺空間思維過程。研究結果發現在非正式的數學學習環境中，使學生能夠展現更多的視知覺過程。

關鍵字:幾何圖形、視覺空間思維、非正式數學學習環境

國小六年級擬題的數學創造力表現之研究

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摘要

本研究旨在探討台灣國小六年級學生在數學擬題任務中的數學創造力表現，根據 Leikin 的創造力評估模型，採用流暢性、靈活性、獨創性三項指標來評估學生的數學創造力(Leikin et al., 2009)。本研究對象為桃園地區國小六年級學生，共十位同學參與施測。施測工具為多元擬題任務，分為三大主題，包含零錢問題、分數問題與成分問題，主要評估學生在日常生活情境中運用數學知識進行擬題的表現。本研究以統計分析為主，輔以訪談，比較學生在不同題目情境下的表現差異。

研究結果顯示，學生在這三項擬題任務中的流暢性表現大致相當，在短時間內產生多個數學問題的能力較為平均。然而，在靈活性與獨創性方面，學生的表現則會因題目條件的限制程度而有所不同。例如，在條件較為開放的情境下，學生能夠提出更多樣化的數學問題，展現出更高的創造力。而當題目條件較為明確且限制性高時，學生的靈活性與獨創性表現則相對受限。

透過學生擬定的題目，研究者能辨識其數學概念中不足，突顯擬題活動可幫助教師診斷學生的數學理解情況。本研究期許擬題活動能促進學生的深度思考與靈活運用，培養數學創造力與解決問題能力，同時為數學教育提供新的評量視角，強調概念建構與創意思維，最終提升學習品質與樂趣。

關鍵字：擬題、創造力、多元評量

結合提問和數學感提升學生文字題解題表現之個案研究

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摘要

本研究旨在探究結合提問策略與數學感學習策略，對國小學生數學文字題解題成效之影響。研究者以基隆市一所非山非市公立小學之四年級三位數學學習扶助學生為研究對象，採用個案研究法進行教學介入與成效分析。教學設計結合提問四策略——複述、回應、追問、挑戰，以及數學感五學習策略——舉例、簡化、畫圖、問為什麼、回想，協助學生建立四步驟解題流程——理解題意、規劃策略、記錄算式、確認答案，藉此引導學生理解題意、建構數學概念並提升解題能力。

資料蒐集包含文字題解題前後測、教學觀察紀錄、學習單與學生訪談，並採量化與質性分析方式探討學生學習表現。前測結果顯示，學生在面對單位轉換、關鍵字誤解及計算錯誤等問題時，解題表現不佳。本研究尚處於實驗初期，僅完成前測與部分觀察分析。

本研究期望能為教學現場提供具實證基礎之教學模式，並強化教師在數學文字題教學上的提問設計與策略運用能力，作為後續教學與研究參考。

關鍵字：提問策略、數學感學習策略、數學文字題、解題表現

八年級學生經歷數位數學遊戲學習等差數列的感受

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摘要

本研究旨在探討將數位數學遊戲融入數學課堂，對八年級學生學習等差數列單元之情意表現的感受。以新竹市某所與桃園市某兩所公立國中八年級共六個班級學生為研究對象，將《速戰數決》數位數學遊戲融入於正式課程中。情意表現藉由情意問卷蒐集學生的情意反映、對課堂參與之感受，分別以敘述性統計與歸納法進行量化與質性的資料整理與詮釋。本研究以量化分析為主、質性分析為輔，綜合評估學生對於數位數學遊戲融入課堂的感受。研究結果顯示，(1)情意表現有明顯的正向效益，學生普遍展現出高度的學習動機與參與度。(2)大多數學生肯定此教學方式，並從學習經驗中提供具建設性的教學與遊戲優化建議。建議未來研究可擴大應用情境與教學單元，設計更具挑戰性的任務並提升評量工具的品質；同時，納入長期追蹤與質性資料，以更全面理解數位數學遊戲對學生學習歷程與成效的影響。

關鍵字：數位遊戲式學習、數位遊戲式教學

數學教育相關研究結果在數學課堂實踐情況的調查研究：以差異化教學為例

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摘要

本研究旨在探討臺北市公立國小數學教學中，差異化教學策略在實踐現況及其影響。研究採用問卷調查法，以北部某師資培育大學在職碩士專班教師為樣本，調查內容聚焦三大議題：教師對數學教育相關研究成果的認識程度；文獻中教學設計在實務現場的應用情形；教師在選擇教學方案時所考量的關鍵因素。問卷中的研究工具取材自國內歷年國小數學領域差異化教學文獻，共 21 篇中選取具有代表性的五篇（針對二至六年級）。資料分析據採用簡單敘述統計，開放式問答題則以內容分析法歸納主題。

結果顯示，約 82.1%的教師未曾閱讀過相關研究文獻，僅有約 5.26%的教師表示曾接觸；而在教學實施方面，59%的教師未採用文獻中所述的策略，僅有 4%的教師完全依樣施行，但多數（約 89.44%）教師持有積極嘗試態度，認為可根據班級實際情況做調整。進一步分析表明，教師認為阻礙實施的主要因素包括教學進度壓力、教材準備困難、班級學生差異以及個人時間與專業發展資源的不足。本研究揭示出現場教師對差異化教學文獻認識不足與策略應用不全之現狀，並指出理論與實務之間存在明顯落差。

關鍵字：理論與實務落差、研究應用、差異化教學

數學奠基模組融入國小六年級圓與扇形面積教學之個案研究

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摘要

本研究欲了解國小六年級學生對於數學奠基模組活動融入圓與扇形面積單元之「學習表現」與「學習態度」。研究採用個案研究法，研究者以一個班級的 5 位學生作為個案學生（偏鄉小校），希望透過在課堂中融入數學奠基活動，提升學生的「學習表現」與「學習態度」。為達到研究目的，本研究工具為前、後測試題與學習回饋單，以及對學生的解題進行資料分析，探討學生的學習情形與學習表現。研究結果發現個案學生學習狀況如下：一、課堂融入數學奠基模組活動有助於提升學生學習表現；二、課堂融入數學奠基模組活動有助於提升學生學習態度。透過此發現，希望提供研究者對於探究學童在國小六年級學習圓與扇形面積單元時能有更多面向的參考。

關鍵字：數學奠基模組、圓與扇形面積

六年級個案學生在比與比值及其相關單元的解題信念

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摘要

本研究是採用個案研究法，以研究者任教的國小三名六年級學生為個案研究對象，探究不同程度學生在比與比值及其相關單元的解題信念。本研究透過問卷、訪談等資料進行分析。研究發現高成就學生在解題時，偏好使用計算時間較少的算法，不會拘泥於課本在該單元所教的公式，能靈活運用先前所學過的概念，並與題目作連結，在解題上較具有彈性但對於估算類型的答案，接受度較低；中高成就學生在解題時，偏好使用課本在該單元所教的公式來作答，解題策略上較僵化，喜歡用一個方法來解所有題目，但面對不同算法時，會嘗試去理解每一個算式的意義，只是不見得能看懂，因此學生雖然能熟悉每一單元所要教的基本公式解，但連結不同單元間的概念的能力較弱；低成就學生在解題時，一樣偏好使用課本在該單元所教的公式來作答，因為學生認為這是一定可以算出正確答案的方法。面對不同算法，雖然接受度是最廣的，但不太會去做深入的思考，因此不見得是真正的理解，考試時也可能完全不會想到這樣的算法，他的認同可能只是來自於覺得老師不會教授錯誤的方法。

關鍵字:比與比值、個案研究、解題信念

國小六年級學生錯誤類型之研究—縮圖、放大圖與比例尺為例

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摘要

本研究旨在分析國小六年級學生，在縮圖、放大圖與比例尺單元中的錯誤類型。研究對象為臺中市兩所國小的 70 名六年級學生，於教學後進行本單元之紙筆測驗，研究者透過試卷結果分析學生答題情形，篩選答對率低於 0.40 之試題，並歸納錯誤類型。研究結果指出，學生的錯誤類型可分為：「概念錯誤」、「表達方式錯誤」與「計算錯誤」。

根據本研究結果，分析學生易混淆的概念有：1.縮圖與原圖之間的邊長關係；2.誤解比例尺的意義。因此，未來教學應提供多樣化的範例與實作活動，例如：可透過數位工具或動畫模擬不同縮放比例的變化，使學生更直觀感受縮圖與原圖的對應關係，輔以測量邊長的練習，以強化學生對縮放與比例尺概念的理解與應用。

關鍵字：比例尺、放大圖、錯誤類型、縮圖

提升四年級學習扶助學生文字題解題能力之個案研究

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摘要

本研究旨在提升國小四年級學習扶助學生解決數學文字題的能力，涵蓋加減法單步驟與兩步驟文字題、乘除法單步驟文字題，以及加減混合乘除的兩步驟文字題。研究者結合多種解題歷程模式與自身的數學教學經驗，設計多元的解題課程，期望協助學生克服學習困難，強化對文字題的閱讀理解與表徵轉換能力，同時提升學習動機與自信，並藉由教學實踐促進自身專業成長。

本研究採用個案研究法，以研究者任教之學習扶助班級中 8 位學生為對象，實施四次聚焦不同題型的教學活動，內容涵蓋文字題解題能力與基本運算能力的培養。在研究過程中，研究者全面記錄並分析學生的學習表現與態度轉變，以探討教學成效。期盼本研究結果能為數學補救教學提供具體可行的策略與實施建議，作為日後教學改進與研究的參考依據。

關鍵字：數學文字題、學習扶助、個案研究法、補救教學

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運用 Polya 解題策略教學降低五年級學生數學學習焦慮之行動研究-以臺中市某國小為例

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摘要

本研究探討 Polya 解題策略對國小五年級學生數學焦慮的影響,採用行動研究方法，透過教學實踐驗證其對降低學生數學焦慮的有效性。數學焦慮會影響學生的學習態度與成就，而 Polya 四步解題法（理解問題、擬定計畫、執行計畫、回顧反思）能幫助學生掌握解題流程，培養有條理的數學思維，提高解決問題的自主性，並增強學生面對挑戰的耐心與信心。

本研究以台中市某國小五年級 23 名學生為對象,透過數學焦慮量表前後測、學習單分析、教師觀察與學生晤談蒐集數據。研究結果顯示，學生的數學焦慮顯著下降，且部分學生表示解題時較不容易緊張，能更有條理地分析問題。在學習成效方面，第六單元測驗成績顯著提升，但第八、九單元未達顯著水準，顯示不同數學概念的學習仍受個別因素影響。學習單分析顯示，多數學生能依循 Polya 策略解題，但「回顧反思」步驟仍需加強。

整體而言，Polya 解題策略可能對減少數學焦慮有所幫助，並可能在部分單元促進學習成效。然而，影響學生學習的因素較為複雜，未來仍需進一步觀察其長期效果。此外，建議教師逐步引導學生熟悉策略，強化「回顧反思」，並適時調整教學方式，以提高學生對解題策略的適應與運用能力。

關鍵字：Polya 解題策略、數學焦慮、行動研究

運用數學寫作於國小五年級整數四則計算之行動研究

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摘要

本研究以臺中市烏日區某公立國小五年級一個班級為研究對象，採行動研究法，探討實施數學寫作時所遭遇的困境與解決策略，以及學生在解釋型、偵錯型與擬題型數學寫作題之答題表現。研究過程中，透過蒐集學生的數學寫作學習單，分析與歸納研究資料，獲得以下研究結果。

在數學寫作活動的實施過程中，教師面臨多項挑戰。首先，低成就學生常不知如何下筆，為解決此問題，教師在日常課堂中融入數學寫作教學，並於寫作過程中鼓勵學生。其次，寫作活動占用較多課堂時間，因此，教師透過綜合課與彈性課來進行數學寫作，以減少對正課的影響。此外，學生對寫作活動的興趣可能隨時間下降，為提升學習動機，教師需調整自身與學生的心態，使數學寫作活動更具吸引力。最後，學生的書寫內容過於簡略，為改善此現象，教師於寫作前展示優秀範例，並提醒學生寫作時應再次閱讀自己的內容，以確保表達完整。

在解釋型、偵錯型與擬題型三種類型的數學寫作題中，學生普遍認為偵錯型題目最為簡單，因其可直接判斷對錯；解釋型題目則因與課堂學習內容相近，學生多能列出正確算式並加以說明，惟部分學生在闡述算式意義時不夠清晰；相較之下，擬題型數學寫作題對學生而言最具挑戰性，高、中成就學生常出現題目情境不適當或表達不完整的問題，而低成就學生則普遍面臨表達不完整、與題意不符或無法作答的情形。

本研究結果顯示，數學寫作雖能提升學生對數學概念的理解與表達能力，然而仍需教師適時調整教學策略，以克服學生在數學寫作活動中可能面臨的困難。

關鍵字：數學寫作、行動研究、解釋型數學寫作、偵錯型數學寫作、擬題型數學寫作

國小高年級學生數學情緒初探

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摘要

本研究旨在發展並驗證國小高年級學生的數學情緒量表，並探討學生在不同背景變項下的數學情緒差異。數學情緒指學生在數學學習、解題及評量時所產生的情緒，影響其動機與學習表現。Pekrun(2006)的控制-價值理論指出，情緒受學生對任務的控制感與價值感影響，進而影響學習態度、動機與成績。因此，本研究欲發展適用於國小高年級學生的數學情緒量表，並分析情緒表現，以及不同背景下數學情緒差異情形。本研究共 868 位國小高年級學生進行施測，本研究設計之數學情緒量表，共分為快樂、得意、焦慮、憤怒、無聊五個向度，使用 Likert 五點量表進行測量。根據結果顯示：(一)量表信度值為.767，具良好信度；(二)因素分析共萃取出五個因素，符合量表設計的五個向度，可解釋總變異量達 73.168%，具良好效度；(三)不同性別之國小高年級學生在數學情緒，快樂、焦慮、憤怒、無聊具有顯著差異，且在負面情緒焦慮、憤怒、無聊中，國小高年級女生的平均數皆高於男生，而在正向情緒快樂中，國小高年級女生的平均數則是低於男生；不同年級之國小高年級學生在數學情緒中，則是均未達顯著差異。本研究結果可為國小數學教學提供參考，幫助教師設計更有效的教學策略，減少學生的負面情緒並提升學習動機。

關鍵字：數學情意、數學情緒量表

數學奠基模組融入國小四年級「假分數和帶分數互換」 教學之個案研究

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摘要

本研究旨在探討奠基模組對學生數學學習成效與學習態度的影響，採用個案研究法，以苗栗縣某國小四年級學童為研究對象。研究結果顯示，學童在參與奠基活動融入課程後，專注力顯著提升，學習動機被有效激發，且能透過不斷的嘗試與修正，逐步掌握假帶分數互換的方法，使整數分割的概念更加清晰與穩固。此外，奠基活動不僅讓學生在互動與探索中深化對分數的理解，也獲得學童正向回饋，顯示其對數學學習的態度更加積極，數學能力亦有所提升。

關鍵字：奠基模組、國小分數、學習成效

以探究教學提升國小五年級學生數學推理與系統性思考之研究

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摘要

本研究旨在探討教師如何運用探究教學，引導國小五年級學生透過探究學習發展數學推理與系統性思考能力。研究採個案研究法，以研究者任教班級六位學生為對象，以「倍數與因數」及「公倍數與公因數」單元進行探究學習，並觀察學生在探究學習歷程中的推理與系統性思考表現。研究結果發現，學生在探究學習歷程中展現不同層次的推理能力：程度較佳學生能快速辨識規律並進行推理擴充，中等學生在引導下能延伸思考，待加強學生則須額外鷹架以進行數學探究。此外，學生透過鷹架設計、顏色提示與操作活動，能更有系統地觀察、比較與歸納數學規律，逐步培養初步的系統性思考能力，並激發學習興趣。學生在數學課堂中不再僅關注計算，而是能主動參與討論與探究數學規律，進而提升數學素養與探究能力。本研究結果可作為實施素養導向與探究教學之參考，並提供教師於高年級數學課程設計上的具體依據。

關鍵字：探究教學、探究學習、數學推理、系統性思考、數學素養

以《圖騰島》桌遊探討小學生空間推理能力之促進效果

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摘要

本研究以專為訓練空間思考能力設計的桌遊《圖騰島》為工具，探討其對學生空間視圖與換位思考能力的影響，並分析遊戲材料的色彩設計對學生學習表現、認知負荷與動機的影響。研究採混合實驗設計，對象為台北市某國小四、五年級共 130 名學生。自變項為遊戲材料色彩組合（單單組：單色目標＋單色積木；單彩組：單色目標＋彩色積木；彩彩組：彩色目標＋彩色積木）與年級；依變項為視圖測驗成績（前測與後測）、認知負荷、學習動機與遊戲喜好。活動後進行訪談蒐集學生回饋。結果顯示，所有年級學生視圖能力皆顯著提升，效果量大（$\eta p^2 = .503$）。彩色材料雖增加進階活動難度，但認知負荷仍屬可接受範圍，且各組在動機與遊戲喜好上皆呈正向反應。學生普遍認為《圖騰島》能提升空間思考、策略與換位思考能力，是具挑戰性與趣味性的學習型桌遊。

關鍵字：空間推理、視圖與視角轉換、桌遊、小學、色彩設計

診斷教學應用於數學分數單元教學成效之研究

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摘要

本研究目的旨在探討診斷教學應用於國小四年級數學分數單元教學之成效。本研究探詮釋性研究法。研究對象為四年級 24 位學生。分數單元內容包括帶、假分數及其互換、大小比較。研究過程先文獻探討相關迷思概念，再設計評量試題診斷學生的迷思概念，進行診斷教學，再探討教學成效。資料收集與分析包括前、後測問卷，教學錄影。信效度採內容效度、專家效度與三角校正。本文僅報導學生的學習成效。研究結果發現學生在帶分數的定義及判別的表現平均得分從 0.08 分(4%)進步到 1.13 分(56.5%)，同分母帶分數與假分數比大小的表現平均得分從 0.5 分(25%)進步到 1.71 分(85.5%)，同分母分數加法的表現平均得分從 1.63 分(81.5%)進度到 2 分(100%)，同分母分數減法的表現平均得分從 1.29 分(64.5%)進度到 1.67 分(83.5%)，帶分數的整數倍計算的表現平均得分從 0.08 分(4%)進步到 1.58 分(79%)。以成績總分 28 分，學生平均得分從 11.7 分(41.8%)進步到平均得分 17.8 分(63.6%)，整體表現有進步。

關鍵字：診斷式教學、分數單元、迷思概念、錯誤類型

異分母分數加減的迷思概念探究與教學設計

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摘要

本研究旨在探討二位國小數學低成就學童在異分母分數加減之迷思概念，並據以設計學習扶助的教學活動。本研究採個案研究，以雲林縣某偏遠地區小學六年級學習扶助班 2 名學童為研究對象，藉由學習扶助前的測驗與晤談，研究發現二位低成就學童在異分母分數加減的迷思概念為:缺乏部分與整體的概念、常誤判單位量、多重表徵轉換困難，難以在文字、圖形與數字間靈活切換等；此外，個案學童在通分時，會習慣性地直接將分母相乘，而非尋找最小公倍數，導致計算負擔增加。為改善上述問題，本研究嘗試設計了結合多元表徵與故事情境的教學活動，希望能提升學童對異分母分數加減的理解。未來將進一步實施教學，同時評估學習成效並提出具體的教學建議。

關鍵字：低成就學童、異分母分數的加減與比較、迷思概念、教學設計

應用四階段評量診斷六年級學童在分數除法問題的表現

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摘要

本研究旨在發展四階段評量（four-tier assessment）診斷工具，探討小六學童在「分數除法」之解題表現及後設認知表現。主要目的分別為發展適用於國小學童在分數除法的四階段評量試題，應用建構反應題評分，分析學童在分數除法之解題類型與表現，並探討學童在分數除法解題之後設認知表現。

本研究之研究對象為國民小學六年級學童，12 個班級進行施測。研究結果發現：(一) 本測驗試卷結果以 Cronbach’s Alpha 係數的信度為第一階段題目係數為.621；第三階段題目係數為.800，顯示本測驗試卷為信度良好試卷。(二)用 Pearson 積差相關來分析鑑別度，所有試題相關係數皆達顯著水準(<.05) 。(三)事實性知識判斷能力方面：研究結果顯示「等組型-帶分數除以帶分數」，答對率為.29，當分數除法無法約分時，學童出現計算錯誤機率較高；其次，「比例型-帶分數除以真分數」，答對率為.39，顯示學童未能將餘數進行單位換算。（四）詮釋性知識的解釋能力方面：第三階段研究結果顯示「等組型-帶分數除以帶分數」平均得分最低，只有.67 分，學童對於數字複雜的分數除法信心較低。其次，「等組型-真分數除以帶分數」與「比例型-真分數除以帶分數」，得分是.73，兩題類型皆是真分數除以帶分數，學童混淆單位量與單位數導致計算錯誤。

關鍵字：分數除法、建構反應題、四階段評量、後設認知

學生影片製作與討論促進數學參與：比較 YouTuber 型式與教學型式任務設計

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摘要

隨著數位媒體日益融入課堂教學，影片製作為促進學生參與數學學習提供了嶄新的契機。先前的研究指出，製作與數學相關的影片有助於提升學生的主動性與參與度，然而目前對於影片製作風格如何影響學生的數學參與仍知之甚少。本研究探討國小學生在兩種對比設計環境中如何參與數學想法的討論：一為 YouTuber 風格的影片任務，另一為教學風格的影片任務。本研究結合多模態分析與數學參與框架，分析學生的對話內容、數學策略使用情形以及互動模式。研究結果顯示，參與教學風格影片任務的學生展現出較高程度的數學參與，相較於參與 YouTuber 風格影片任務的學生更具成效。本研究有助於拓展關於影片媒介在數學討論中應用的相關文獻，並提供設計具有文化回應性且結合媒體素養之數學學習經驗的實務建議。

關鍵字: 影片製作、線上影音平台、學生數學參與度

因倍數概念之學習診斷工具開發之前導研究

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摘要

國中小階段的因數與倍數概念是分數運算與解方程式的基礎，但學生常因概念不清，面臨解題困難。研究發現，學生在學習此單元時，常遺漏因數、混淆概念、誤用策略，甚至放棄應用題，產生學習無助感。為此，研究者針對國小五年級至國中八年級學生，參考因材網知識結構星空圖，開發一套診斷工具，幫助教師快速識別學生薄弱點。教師可透過診斷，在教學前針對性複習，強化先備知識，提升學習成效，減少因概念不清導致的困難。

關鍵字：因倍數概念、先備知識、知識結構、學習診斷

應用三階段評量探討國小五年級學童在因倍數問題的解題表現

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摘要

本研究旨在發展三階段評量診斷工具，據以探討國小五年級學童在因數與倍數問題上的迷思概念(misconception)，分析學童在因數、倍數問題的解題表現和類型，並探討和後設認知表現之相關。本研究以 215 位來自不同地區的國民小學五年級學童為對象，研究工具採用自編國小五年級因數、倍數的三階段評量。研究結果顯示：(一)本測驗試卷結果以 Cronbach's Alpha 係數的信度係數結果為.677 及.854，顯示本測驗試卷為信度良好試卷。(二)用 Pearson 積差相關分析試題鑑別度，所有試題相關係數皆達顯著水準(<.05)，表示試題能區辨學生的答題能力。(三)事實性知識判斷能力方面：答對率最低的錯誤類型是「尋找倍數時，出現遺漏本身之情形」，答對率為.34，顯示學童易忽略每個數自身即為最小倍數。（四）詮釋性知識的解釋能力方面：學童在「尋找因數時，出現遺漏 1、本身或部份因數之情形」平均得分最低，只有.70 分，顯示其在解釋時容易忽略從最小因數 1 開始列舉或未完整找出所有因數。整體而言，本研究的評量工具具備良好的信度與鑑別度，可作為未來探討國小學童因數與倍數概念理解的參考依據。

關鍵字：因數、倍數、三階段評量、建構反應題、後設認知

差異化教學中的彈性分組策略對國小六年級學生數學學習成效之影響

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摘要

本研究旨在探討差異化教學中的彈性分組策略對於六年級學生對數學學習成效的影響。差異化教學強調根據學生的學習準備度、興趣與學習風格設計多樣化的教學內容與活動，以提升全體學生的學習表現；彈性分組則透過動態調整小組組成，促進同儕合作與資源共享，以滿足不同程度學生的學習需求。本研究以準實驗研究法進行研究，設計具體的教學活動，並結合多元評量方式，評估彈性分組策略在「角柱與圓柱」單元（本單元同時包含表面積與體積的計算）中的學習成效與學習態度影響。

本研究對象為研究者任教學校的六年級學生，採用不等組前後測設計（Nonequivalent Control Group Pretest-Posttest Design），選取兩個班級進行實驗。實驗組採用彈性分組策略，對照組則維持原教學模式。研究工具包括數學學習成效測驗、學習態度與學習動機問卷，並透過 t 檢定與共變數分析（ANCOVA）進行統計檢定，以評估教學策略的效果。

關鍵字：差異化教學、彈性分組、數學學習成效

探究教師在數學數位差異化教學中轉變之理論框架

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摘要

隨著教育科技的快速發展，將數位工具融入數學教學已成為滿足學生多樣化學習需求的重要方法 (Drijvers, 2020; Hwang et al., 2021) 。數位差異化教學應運而生，旨在通過利用數位工具提供學生個性化的學習路徑 (Tomlinson, 2017) 。教師在適應教育改革的過程中需要經歷複雜的轉變歷程 (Lawless & Pellegrino, 2007) ，並受到個體知識信念、機構支持和社會文化背景的共同影響(Ertmer & Ottenbreit-Leftwich, 2013) 。現有對教師的轉變的研究主要關注教師自評知能、態度與外顯行為之變化，而在解釋教師轉變的方面缺少深度，教師轉變的內在機制如何運作常被忽略。

鑑於轉化學習理論 (Transformative learning theory, TLT) (Mezirow, 2008)強調個體對於關鍵事件的批判反思，有助於解釋教師在數位差異化教學中所經歷的循環往復、深層次與根本性的轉變；以及關注本位採用模式 (Concerns-Based Adoption Model, CBAM)　(Hall & Hord, 2015)提供結構化模型，可用於刻化教師在數位差異化教學過程中所呈現的關注階段和使用層次。基於這兩個理論視角的互補性，並考量教師轉變受個體差異及環境因素的交互影響，本研究整合 TLT 與 CBAM 形成理論框架，旨在探究數位差異化教學中教師在個體因素與環境脈絡交織下所展現的顯性與隱性轉變，並進一步剖析個體轉變所涉及的內在機制。本框架不僅有助於闡明數學教師在數位差異化教學環境下的挑戰與良機，還能提供具體的理論支撐以探討教師轉變歷程中的關鍵因素。

關鍵字：教師轉變、轉化學習理論、關注本位採用模式、數位差異化教學

國小二年級低成就學童數學外加課程舉例能力之教學初探

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摘要

本研究目的為探究二年級低成就學童在一個學期融入「讓學生舉例」的外加課程中，其舉例能力的改變情形與改變原因。研究方法採個案研究法，研究對象為三名數學低成就學生。研究過程為在一學期十個單元的內容中，利用每週一節彈性時間的外加課程，對在原班級已經學過的內容，利用學習單融入舉例的問題，培養學生對各單元某概念、內容的舉例能力，同時針對其舉例內容加以引導。資料收集包括學習單、教學錄影、學生訪談，資料的信、效度採用內容效度、專家效度與三角校正法。本研究僅報導整數與運算單元的內容。

研究發現學童一開始僅能簡單的寫出幾個字，到後來能較完整的寫出二步驟加減運算的例子。學童改變的原因在於學生在安全的學習環境中，有機會口述他們的想法，寫下他們的想法。即使他們說得不完整，教師仍加以鼓勵，強化學生的自信心。

關鍵字：舉例、外加課程、低成就學童、國小二年級數學

國中數學課後學習扶助使用數位學習平台之個案研究

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摘要

本研究採用個案研究法，旨在探討數位學習平台對國中數學課後學習扶助的影響，研究焦點聚焦於八年級上學期「因式分解」單元。研究對象為台中某私立國中一名數學低成就學生。透過前測、後測、半結構式訪談與反思日誌分析學生的學習歷程與成效。研究結果顯示，個案在使用數位學習平台後，掌握了提公因式與十字交乘法等關鍵概念。其後測的答對率由原先的 38%提升至 72%，顯示個案學生有明顯的進步。數位平台中的教學影片與即時回饋機制，有效協助學生釐清迷思概念與提升學習信心。研究建議可將此平台應用至其他數學單元與不同學生群體，以進一步檢驗其普適性與成效，日後可作為補救教學策略設計與數位工具整合的實務參考。

關鍵詞：數位學習平台、因式分解、數學學習扶助、學習歷程

學生小組成就區分合作學習融入六年級小數與分數的計算教學之研究

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摘要

本研究旨在探討小組成就區分合作學習法（STAD）對國小六年級學童在「小數與分數的計算」單元之學習成就與學習動機的影響，本研究採用實驗研究法分析兩組學生在數學學習成就上的差異，並探討實驗組學生在學習動機前後測之變化，期望研究結果能為國小教師在數學教學實踐中提供參考。選取雲林縣偏遠地區麥寮鄉兩所國小作為研究場域，參與學生分配至實驗組與對照組，研究者參考相關文獻，設計並實施適用於實驗組的合作學習教學實驗。在量化分析方面，本研究蒐集兩組學童的「小數與分數的計算成就評量」成績，並針對實驗組進行「數學學習動機量表」的前後測分析，以檢視不同教學方法對學習成效與學習動機的影響。

在質性分析部分，每次教學結束後，實驗組學童需撰寫數學學習日誌，記錄當日學習收穫與心得。另外學童會以小組為單位進行討論，研究者則透過平板錄音觀察紀錄各組的討論情形。完整教學實驗結束後，研究者依據小數與分數的計算成就評量的高、中、低分組，從實驗組各組選出 2 位(一名男生、一名女生)，進行學習動機相關的個別晤談，以深入了解合作學習對學生學習態度與動機的影響。

結果顯示兩種教學法在學習成效與學習動機上的提升無顯著差異，雖然實驗組學生在學習興趣、自信心與投入度方面有所提升，但與對照組相比，未達顯著性。

關鍵字：小組成就區分法、合作學習、學習動機、偏遠地區

以數學感「舉例」策略融入一年級數的單元之初探

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摘要

本研究目的旨在探討將「舉例」策略融入國小一年級數學數的單元教學之成效與改變原因。為達到研究目的，研究者採用個案研究法，運用舉例練習單分析一年級學童在數的單元舉例能力的變化。研究過程考量學生的書寫能力尚未完善，全班先口頭練習如何舉例、注意量詞，之後再讓學生運用書寫方式進行舉例。資料收集與分析包括學生的學習單，教學錄音、錄影，教學札記。資料分析的信度、效度採用內容效度、專家效度，以及三角校正。研究結果發現上學期能完整舉例的大約四成，到下學期的二十以內的加法進步到七成。學生的改變與老師在教學中時強調生活量詞的正確性，先想一想再說出來，以及找出課本例題的重點來了解重要概念，同時大約在 1 到 2 個月後才讓學生練習舉例，使學生有機會慢慢形成學習習慣，進而內化為學習能力有關。

關鍵字：舉例、數與量、一年級數學

IMPROVE 後設認知教學法對國小學生數學學習影響初探

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摘要

本研究旨在探討運用 IMPROVE 後設認知教學法與後設認知提問，對國小六年級學生數學學習成效的影響。本研究根據 Mevarech 與 Kramarski(1997)提出的 IMPROVE 後設認知教學法以及相關文獻作為理論基礎，採用準實驗研究設計，研究對象以台中某國小六年級兩個班級的學生作為實驗組與對照組。研究者對實驗組實施六堂課之 IMPROVE 後設認知教學法融入最大公因數與最小公倍數單元之教學，對照組實施講述式教學法在同一單元之教學。教學後以翰林版最大公因數與最小公倍數單元測驗卷之測驗成績作為依變項，並以學生五年級下學期數學定期評量成績作為共變項，進行單因子共變數分析，並探討學生在數學文字題的答題錯誤類型差異。本研究結果發現：(一)在經過實驗教學後，實驗組與對照組學生未因不同教學法而在學習成效上有所差異。(二)分析兩組學生在單元測驗卷中數學文字題的答題錯誤類型，發現兩組學生有相似的錯誤概念產生。(三)同樣錯誤類型的答題歷程來看，實驗組學生相較於對照組學生較能關注到題目中給定的條件並作答。

關鍵字：後設認知、IMPROVE 後設認知教學法、後設認知提問

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暨數學教學工作坊摘要

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海報發表

從棋盤到決策：透過棋類遊戲培養學生的樹狀圖思維與決策能力之研究

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摘要

本研究目的旨在探討棋類遊戲如何作為教育工具，培養學生的樹狀圖思維與決策能力。透過文獻回顧與實證分析，研究者發現棋類遊戲能有效引導學生建立系統性的思考，幫助他們在面對問題時，運用層層推演的策略，進行周全的決策評估，而棋類遊戲的規則與機制，促使學生透過預測、分析與應變來做出最佳選擇，從而提升數學推理與問題解決的能力。另外，本研究建議，若能結合其他的互動工具，如數位軟體、程式設計、解謎等活動，將能進一步地擴展學生的學習情境，進行跟深入的研究，使得樹狀圖思維的應用變得更加靈活且多元。

根據研究結果顯示，本研究亦建議教師在課堂上適當的融入棋類遊戲，以啟發學生的決策思維，並結合多種教學工具，創造更具挑戰性與啟發性的學習環境。此外，課程設計應鼓勵學生透過遊戲中的實作與反思來提升決策品質，使其能夠在學科學習上以及日常生活中，靈活的應用樹狀圖思維，培養面對未來挑戰的能力。

關鍵字：棋類遊戲、樹狀圖、決策能力

GeoGebra 融入三次函數對稱中心學習扶助研究

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摘要

本研究主要探討高中一年級學生在「GeoGebra 融入三次函數的對稱中心」前後的學習表現差異。首先利用前測了解學生的錯誤類型，並在課堂活動過程中引入 GeoGebra，配合其動態特性與直觀的圖像表徵，讓學生實際操控變數使其了解對稱中心的意義與三次函數的關係。經由課室觀察與相關資料的蒐集，研究發現個案學生在學習扶助前常見的錯誤類包含：無法理解對稱中心的意義、無法從特定形式的三次函數如 y=ax-h3+px-h+k 看出三次函數的對稱中心、也無法從三次函數的一般式 y=ax3+bx2+cx+d，轉化為 y=ax-h3+px-h+k 看出對稱中心；在學習扶助活動進行中，學生訪談中表示，GGB 的動態性以及具體的圖像觀察三次函數圖形對稱中心有直觀的感受使其印象深刻，在數學學習表現上有明顯興趣提升；且經由 GeoGebra 融入學習扶助後，個案學生在後測檢驗後，對於對稱中心與三次函數的關聯有顯著的改善。

關鍵字: 三次函數，GeoGebra，學習扶助

設計動態虛擬教具支援多重積分的學習

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摘要

多重積分的學習重點在二重和三重積分，而要解決多重積分計算問題就必須要先能掌握積分區域描述的能力。本研究設計的動態虛擬教具暨使用平台(GGB)聚焦在 2-和 3-維區域的探討，旨在輔助多重積分的學習，適用於課堂上教學與課後自我練習之用。「動手寫」是學習數學的不二法門，GGB 營造一個讓學生必須動手演練的具備回饋機制的動態數位學習環境。本文將同時介紹應用於二重積分的 GGB 和應用於三重積分的 GGB。然而，由平面區域的描述跨越到立體區域描述難度是倍數成長，因此，本研究先針對二重積分的學習，透過紙本測驗方式分析 GGB 對學習成效的影響。分析結果顯示，在常態的積分教學導入 GGB 確實對學習有正面影響。

關鍵字：二重積分、多重積分、GeoGebra、迭代積分、動態虛擬教具

國小數學教科書素養內容分析-以分數為例

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摘要

本研究旨在探討十二年國教課綱下，臺灣國小數學教科書分數單元中，如何融入數學素養導向內容，並以 PISA2022 數學推理三階段「形成、應用、詮釋與評鑑」為分析架構，選取市佔率較高之 K、H 和 N 三版本教科書作為研究對象，採用內容分析法，針對教材中相關題型進行分類與統計，探究教科書編排脈絡與素養表現的異同。

研究結果發現：三個版本中皆以「形成」階段比例最高，反映出教科書多著重於觀念建立及反覆計算的訓練；「應用」階段比例次之，顯示部分教材已開始導入問題建構與模型轉化的素養訓練；「詮釋與評鑑」階段比例則偏低，顯示學生反思能力與真實應用的訓練機會有限。此外，不同版本教科書在教材脈絡與素養導向呈現方式上亦各具特色。

根據研究結果，建議教材編寫者強化「詮釋與評鑑」相關題型設計，並提升跨情境與生活應用素材的結合程度；教師亦可依學生需求補充素養導向任務設計，促進學生批判思考與數學應用能力。未來研究可進一步擴大分析對象及數學單元，並結合教學現場觀察與學生回饋，以深化教材分析的實務意義。

關鍵字：國小數學、教科書分析、數學素養、PISA、分數單元

辦理單位

台灣數學教育學會
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指導單位

國家科學及技術委員會
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